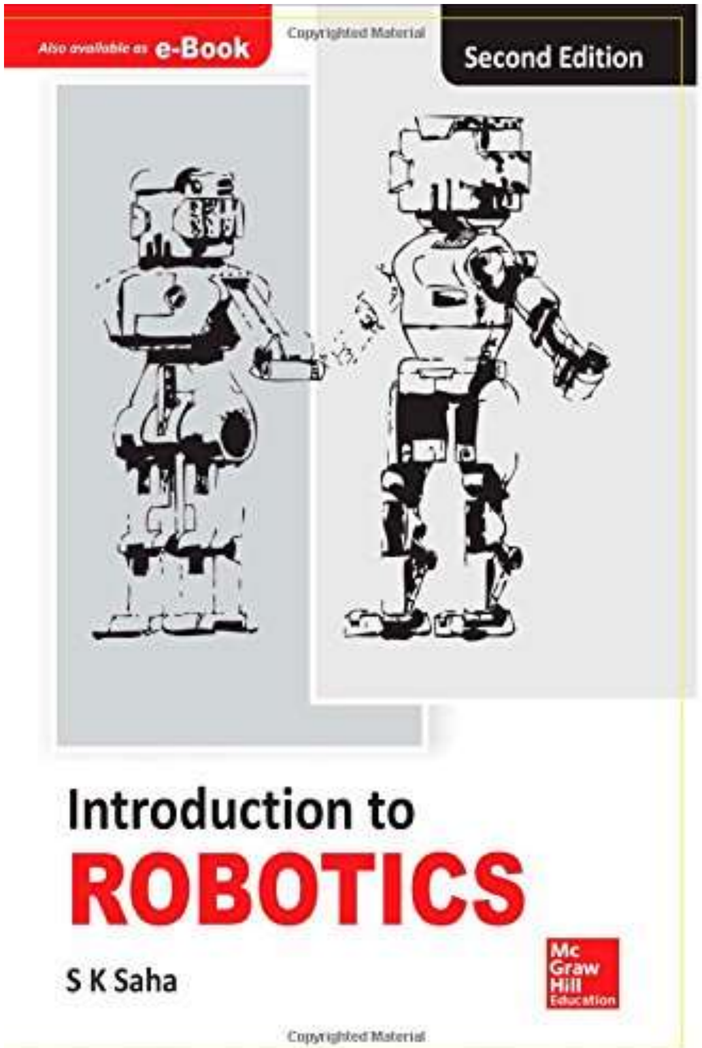
McGraw-Hill Education



Lecture 2 (III-LT1)

Robot Kinematics (Ch. 5)

by

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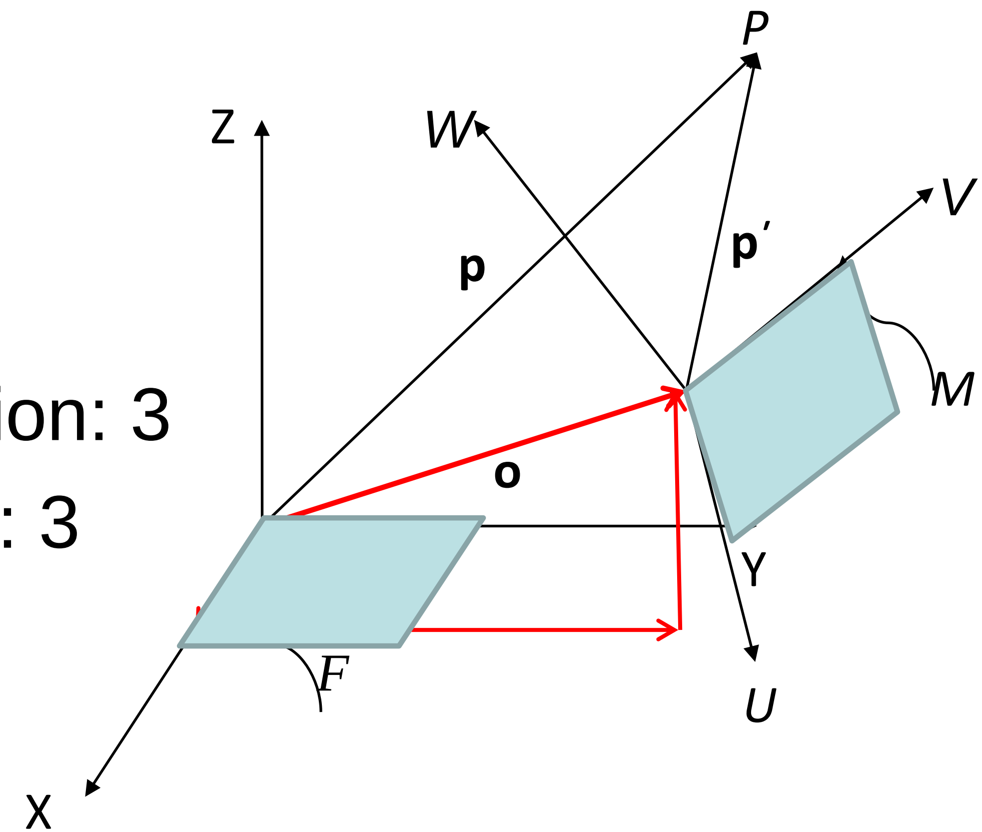
Aug. 07, 2015 (F)@JRL301 (Robotics Tech.)

5

Transformations

Spatial Motion

Pose ≡ Position + Rotation



Translation: 3

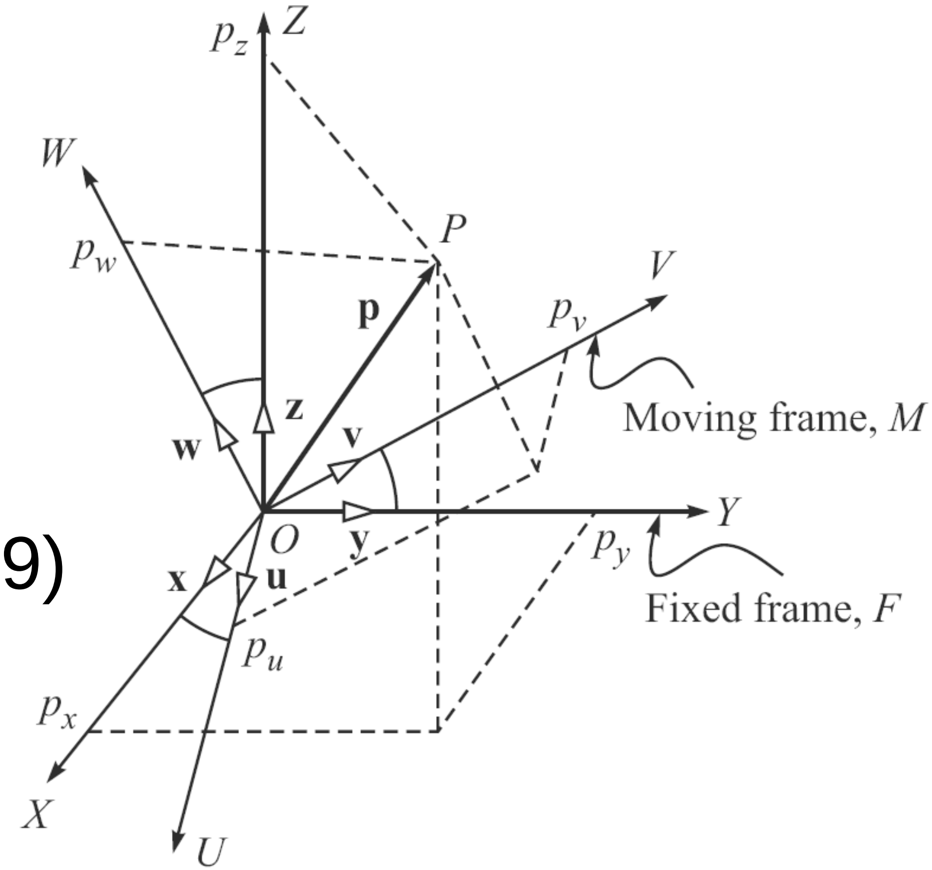
Rotation: 3

Total: 6

A moving body → Pose or Configuration

Position Description

$$[\mathbf{p}]_F \equiv \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \qquad \ldots (5.8)$$



$$\mathbf{p} = p_x\,\mathbf{x} + p_y\,\mathbf{y} + p_z\,\mathbf{z} \qquad \ldots (5.9)$$

Fig. 5.12 Spatial description

$$[\mathbf{x}]_\mathbf{F} \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ [\mathbf{y}]_\mathbf{F} \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \text{and}\ [\mathbf{z}]_\mathbf{F} \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \ldots (5.10)$$

Orientation Description

1. Direction cosine representation

2. Fixed-axes rotations

3. Euler angles representation

4. Single- and double-axes rotations

5. Euler parameters representation

I will illustrate the first TWO only

Direction Cosine Representation

Refer to Fig. 5.12

$$\mathbf{p} = p_u\mathbf{u} + p_v\mathbf{v} + p_w\mathbf{w} \quad \ldots (5.12)$$

$$\mathbf{u} = u_x\,\mathbf{x} + u_y\,\mathbf{y} + u_z\,\mathbf{z} \quad \ldots (5.11a)$$

$$\mathbf{v} = v_x\,\mathbf{x} + v_y\,\mathbf{y} + v_z\,\mathbf{z} \quad \ldots (5.11b)$$

$$\mathbf{w} = w_x\,\mathbf{x} + w_y\,\mathbf{y} + w_z\,\mathbf{z} \quad \ldots (5.11c)$$

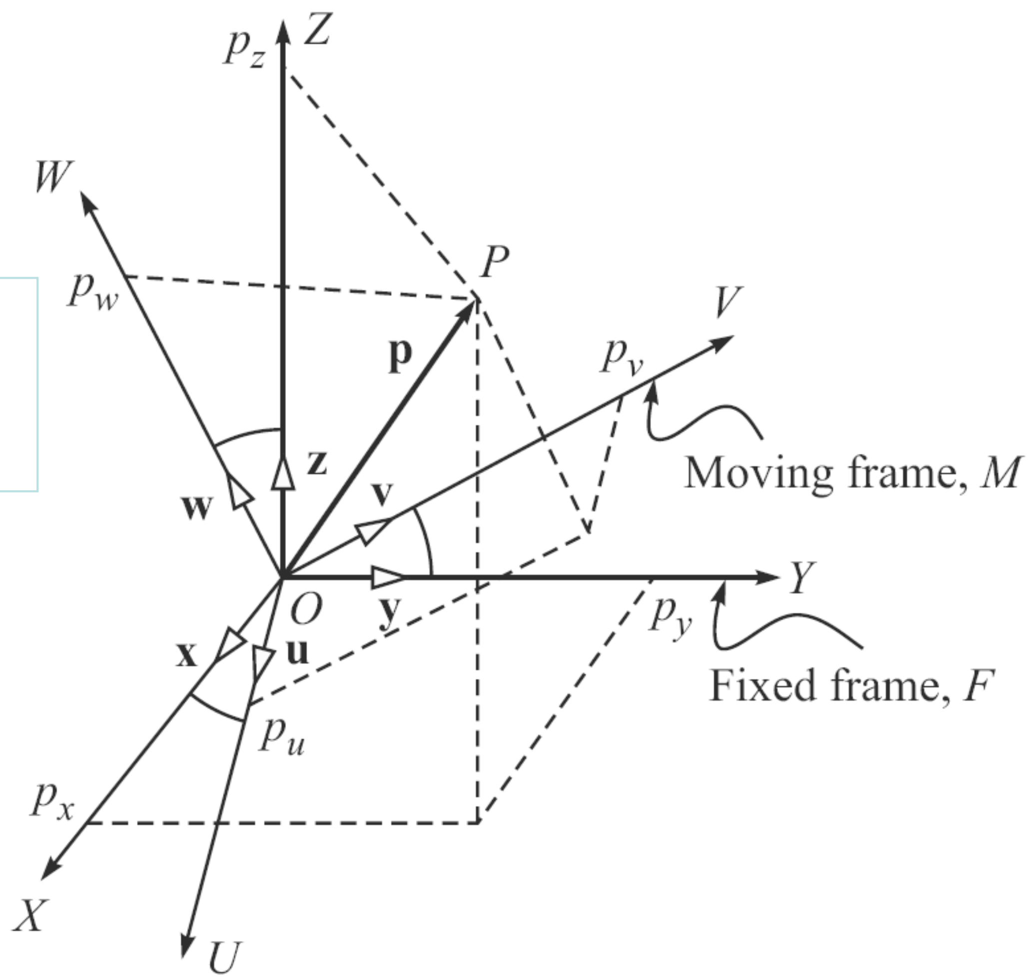


Fig. 5.12 Spatial description

Substitute eqs. (5.11a-c) into eq. (5.12)

$$\mathbf{p} = (p_u u_x + p_v v_x + p_w w_x)\mathbf{x} + (p_u u_y + p_v v_y + p_w w_y)\mathbf{y} + (p_u u_z + p_v v_z + p_w w_z)\mathbf{z} \quad \ldots (5.13)$$

$$p_x = u_x p_u + v_x p_v + w_x p_w \quad \ldots (5.14a)$$

$$p_y = u_y p_u + v_y p_v + w_y p_w \quad \ldots (5.14b)$$

$$p_z = u_z p_u + v_z p_v + w_z p_w \quad \ldots (5.14c)$$

$$[\mathbf{p}]_F = \mathbf{Q}\,[\mathbf{p}]_M \quad \ldots (5.15)$$

$$[\mathbf{p}]_F = \mathbf{Q}\,[\mathbf{p}]_M \qquad \ldots (5.15)$$

$$[\mathbf{p}]_F \equiv \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}, \quad [\mathbf{p}]_M \equiv \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}, \quad \mathbf{Q} \equiv \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = \begin{bmatrix} \mathbf{u}^T\mathbf{x} & \mathbf{v}^T\mathbf{x} & \mathbf{w}^T\mathbf{x} \\ \mathbf{u}^T\mathbf{y} & \mathbf{v}^T\mathbf{y} & \mathbf{w}^T\mathbf{y} \\ \mathbf{u}^T\mathbf{z} & \mathbf{v}^T\mathbf{z} & \mathbf{w}^T\mathbf{z} \end{bmatrix} \qquad \ldots (5.16)$$

Orientation description 1

$\mathbf{u}^T\mathbf{u} = \mathbf{v}^T\mathbf{v} = \mathbf{w}^T\mathbf{w} = 1$, and

$$\mathbf{u}^T\mathbf{v}(\equiv\mathbf{v}^T\mathbf{u}) = \mathbf{u}^T\mathbf{w}(\equiv\mathbf{w}^T\mathbf{u}) = \mathbf{v}^T\mathbf{w}(\equiv\mathbf{w}^T\mathbf{v}) = 0 \qquad \ldots (5.17)$$

Q is called Orthogonal

Due to orthogonality

$$\mathbf{u} \times \mathbf{v} = \mathbf{w}, \quad \mathbf{v} \times \mathbf{w} = \mathbf{u}, \text{ and } \quad \mathbf{w} \times \mathbf{u} = \mathbf{v} \quad \ldots (5.18)$$

$$\mathbf{Q}^T\mathbf{Q} = \mathbf{Q}\mathbf{Q}^T = \mathbf{1}\,; \ \det(\mathbf{Q}) = 1; \ \mathbf{Q}^{-1} = \mathbf{Q}^T \ \ldots (5.19)$$

Example 5.6 Rotations [Elementary] (Fig. 5.13a)

Interpretation 1

$$[\mathbf{u}]_F \equiv \begin{bmatrix} C\alpha \\ S\alpha \\ 0 \end{bmatrix}, \quad [\mathbf{v}]_F \equiv \begin{bmatrix} -S\alpha \\ C\alpha \\ 0 \end{bmatrix}, \quad [\mathbf{w}]_F \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \ldots (5.20)$$

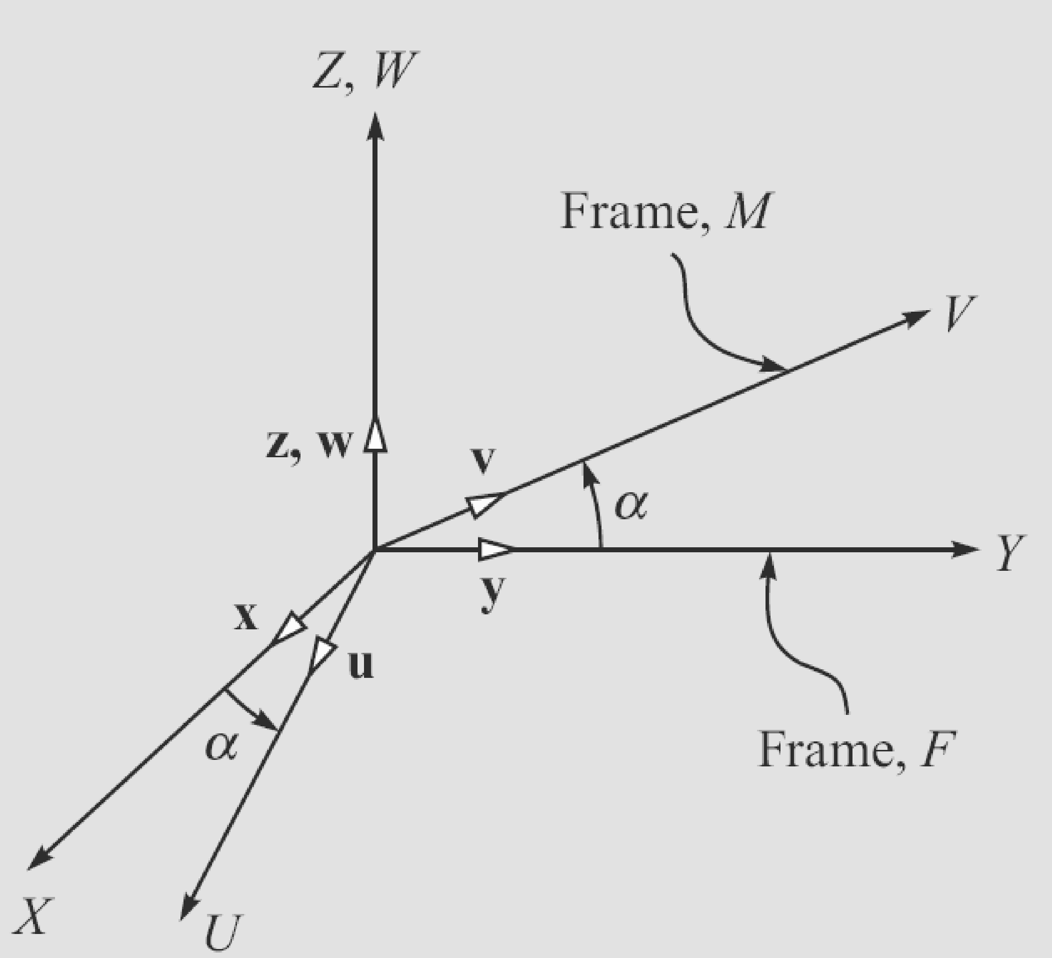


Fig. 5.13 (a) Rotation about *Z*-axis

$$\mathbf{Q}_Z \equiv \begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{. . . (5.21)}$$

$$\mathbf{Q}_Y \equiv \begin{bmatrix} C\beta & 0 & S\beta \\ 0 & 1 & 0 \\ -S\beta & 0 & C\beta \end{bmatrix}; \quad \mathbf{Q}_X \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\gamma & -S\gamma \\ 0 & S\gamma & C\gamma \end{bmatrix}$$

$$\text{. . . (5.22)}$$

Non-commutative Property: An Illustration

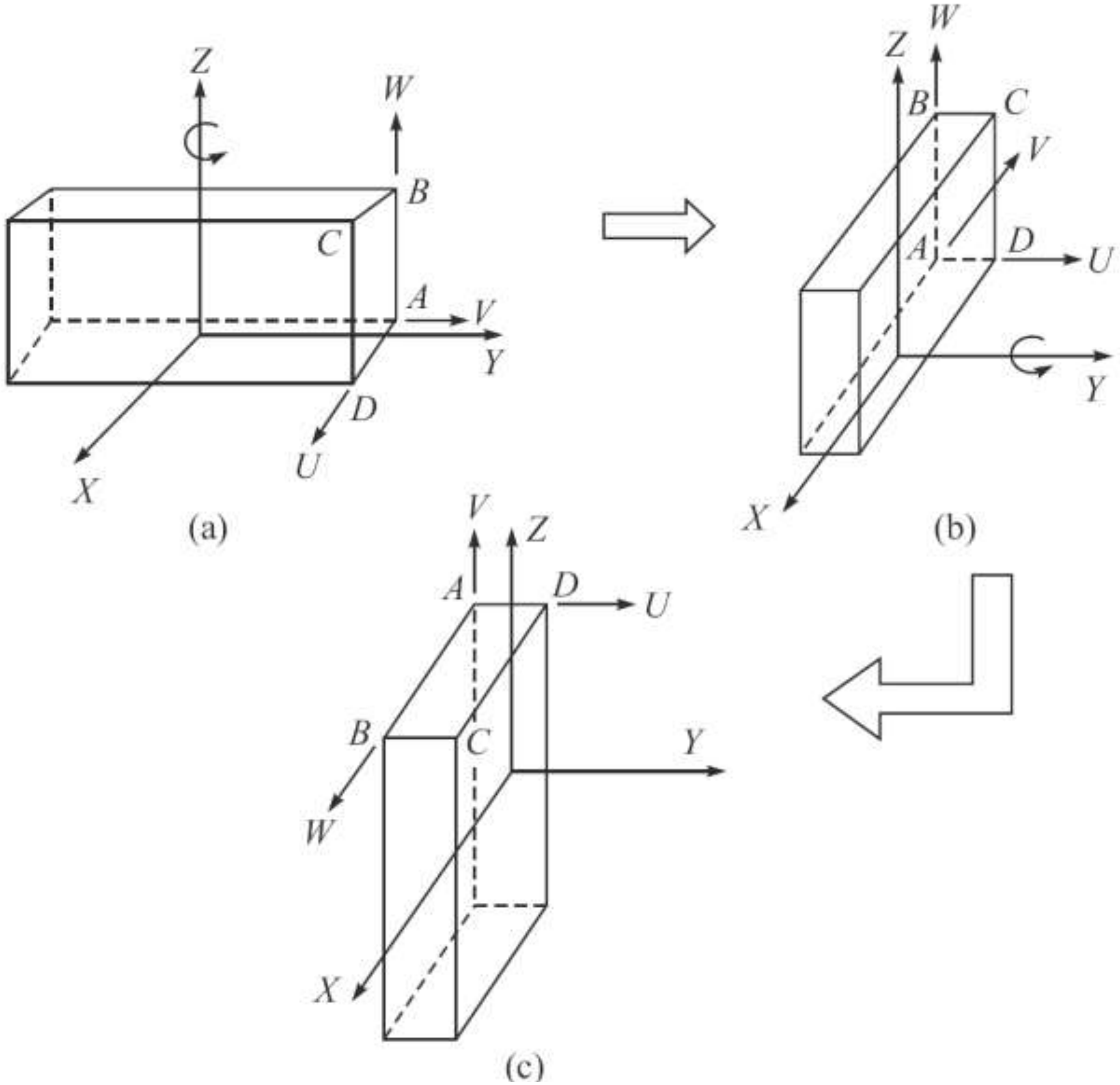


Fig. 5.20 Successive rotation of a box about Z and Y-axes

Non-commutative Property (contd.)

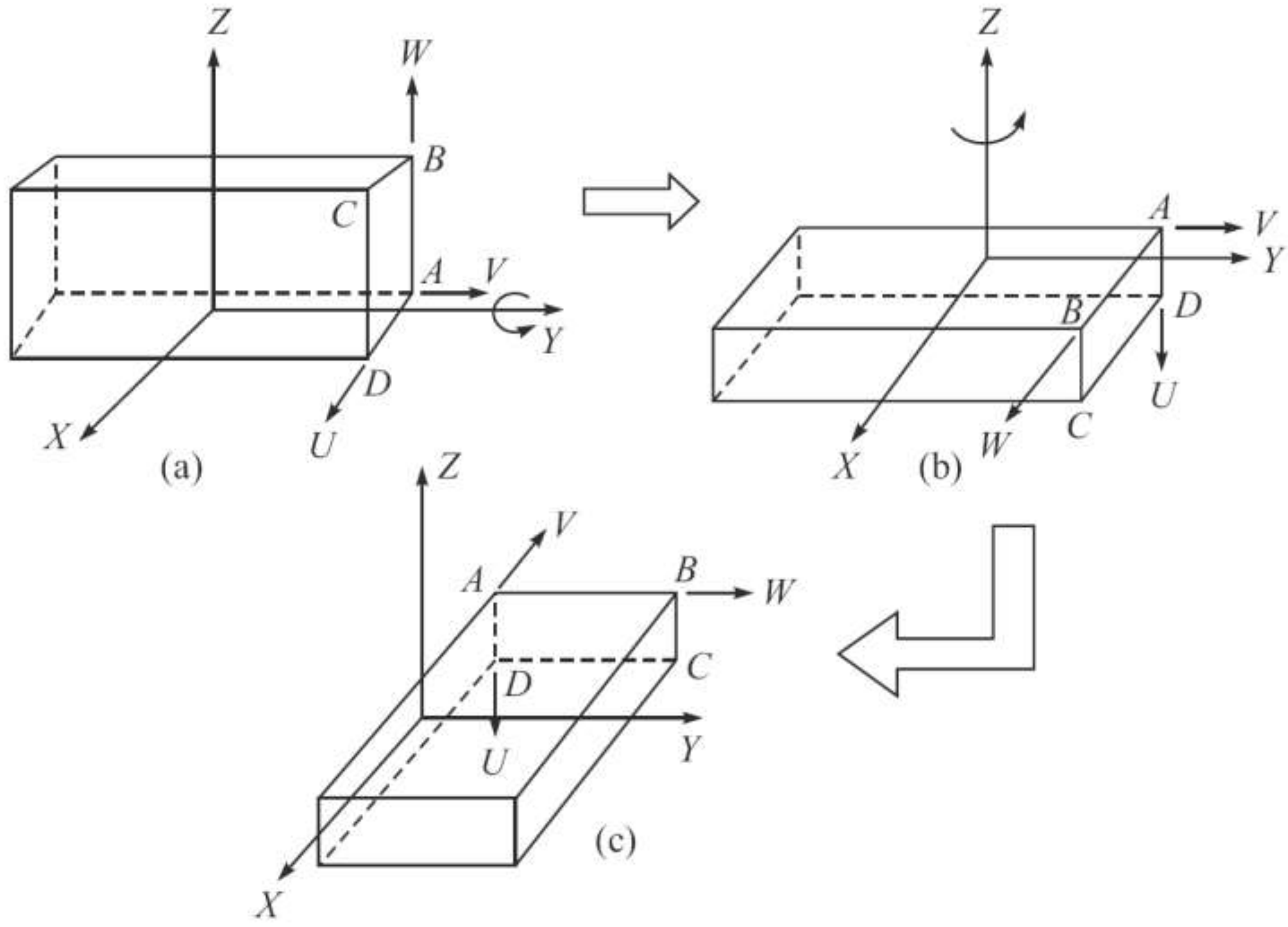
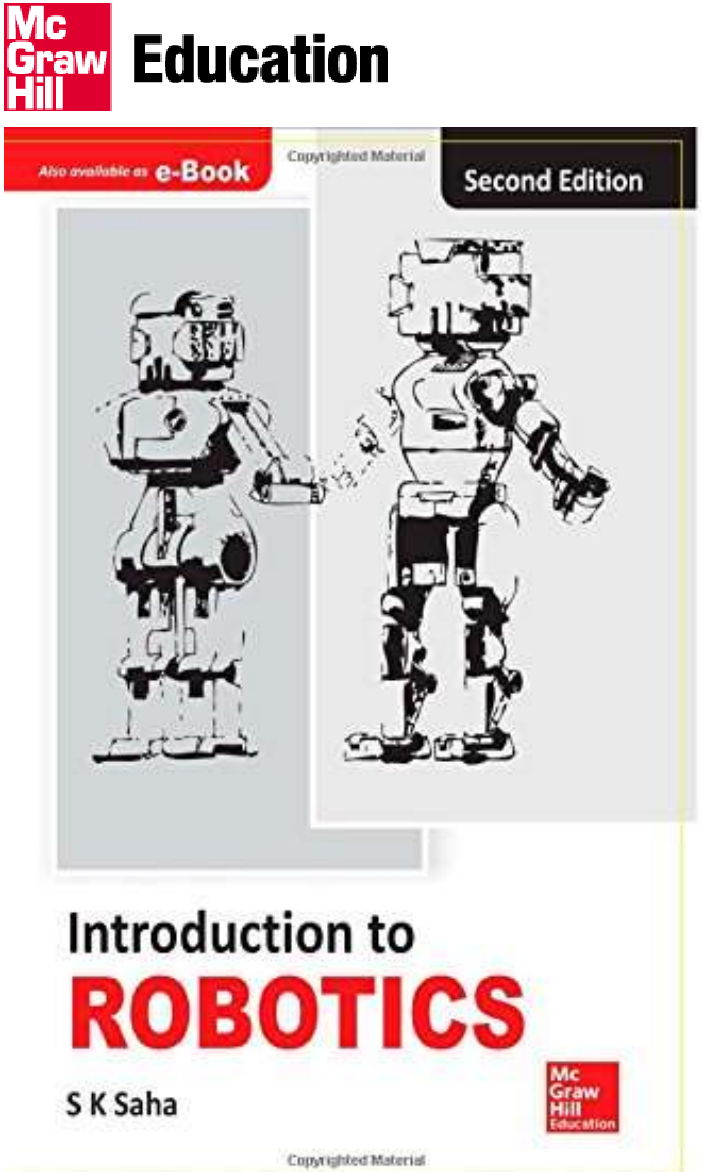


Fig. 5.21 Successive rotation of a box about Y and Z-axes



Lecture 3

Robot Kinematics (Ch. 5)

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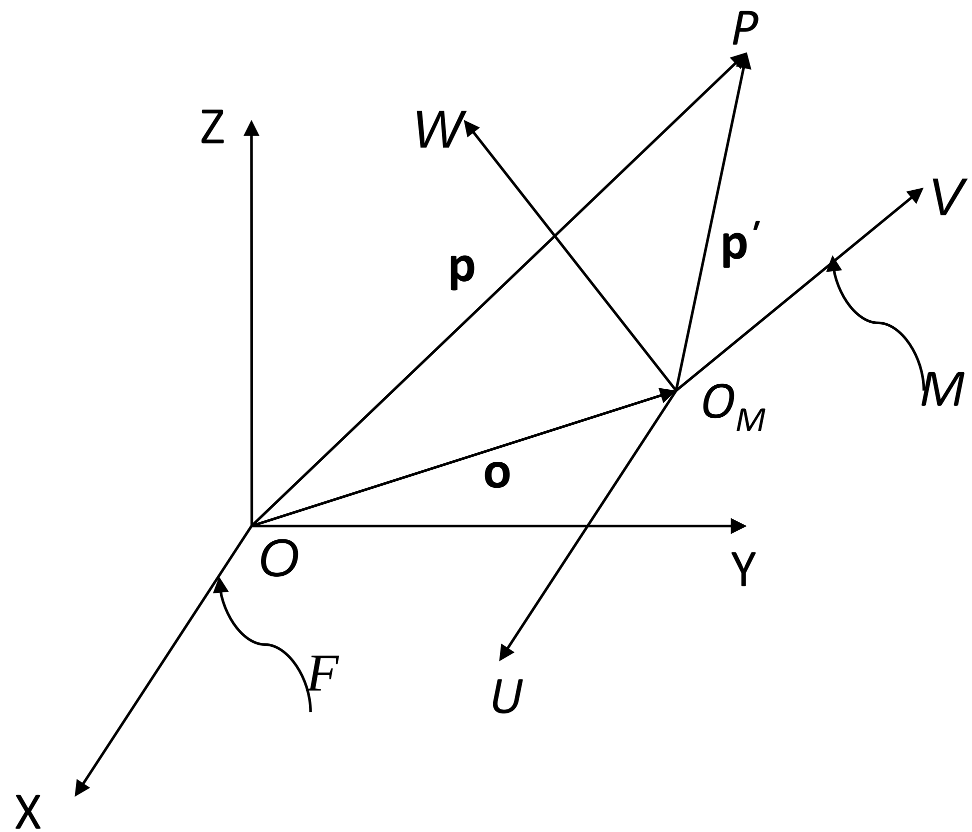
Recap

- Orientation representations
  - Non-commutative
- Direction cosines: Has disadv. of 9 param.
- Fixed-axes (RPY) rotations (12 sets)

Homogeneous Transformation



Task: Point *P* is known in moving frame *M*. Find *P* in fixed frame *F*.

Fig. 5.23 Two coordinate frames

$$\mathbf{p} = \mathbf{o} + \mathbf{p}' \qquad \ldots (5.45)$$

$$[\mathbf{p}]_F = [\mathbf{o}]_F + \mathbf{Q}[\mathbf{p}']_M \qquad \ldots (5.46)$$

$$\begin{bmatrix} [\mathbf{p}]_F \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q} & [\mathbf{o}]_F \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} [\mathbf{p}']_M \\ 1 \end{bmatrix} \qquad \ldots (5.47)$$

$$[\overline{\mathbf{p}}]_F = \mathbf{T}[\overline{\mathbf{p}'}]_M \qquad \ldots (5.48)$$

Homogenous Transformation

T: Homogenous transformation matrix (4 × 4)

$$\mathbf{T}^{\mathrm{T}}\mathbf{T} \neq \mathbf{1} \quad \text{or} \quad \mathbf{T}^{-1} \neq \mathbf{T}^{\mathrm{T}} \qquad \ldots (5.49)$$

$$\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{Q}^{\mathrm{T}} & -\mathbf{Q}^{\mathrm{T}}[\mathbf{o}]_F \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix} \qquad \ldots (5.50)$$

Example 5.10 Pure Translation

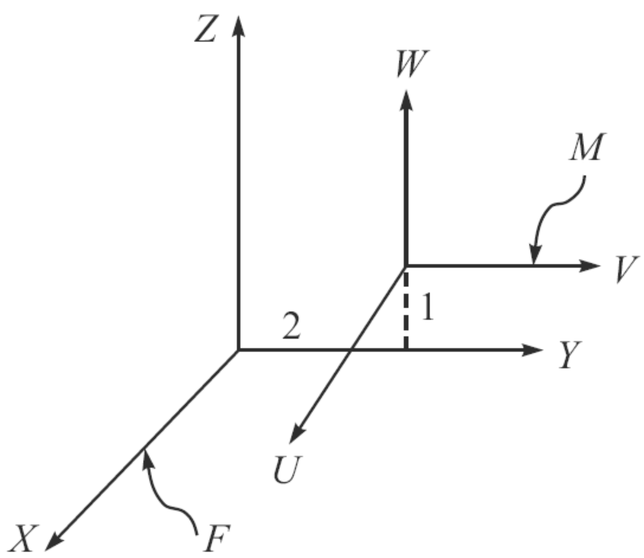


Fig. 5.24 (a)

$$\mathbf{T} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.51)$$

Example 5.11 Pure Rotation

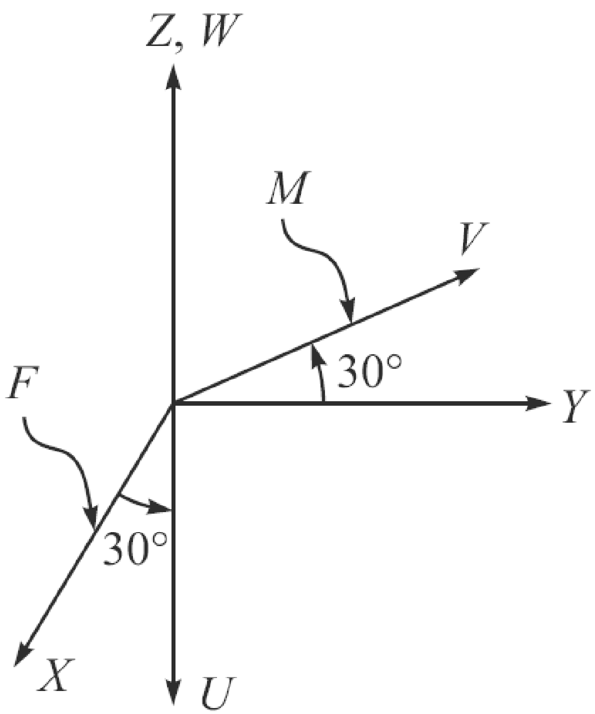


Fig. 5.24 (b)

$$\mathbf{T} \equiv \begin{bmatrix} C30^o & -S30^o & 0 & 0 \\ S30^o & C30^o & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{3}{2}} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.52)$$

Non-commutative Property

Like rotation matrices homogeneous transformation matrices are non-commutative, i. e.,

$$\mathbf{T}_A\mathbf{T}_B \neq \mathbf{T}_B\mathbf{T}_A$$

Denavit and Hartenberg (DH) Parameters

- Serial chain
- Two links connected by revolute or prismatic joint

- Four parameters
  - Joint offset (b)
  - Joint angle (θ)
  - Link length (a)
  - Twist angle (α)

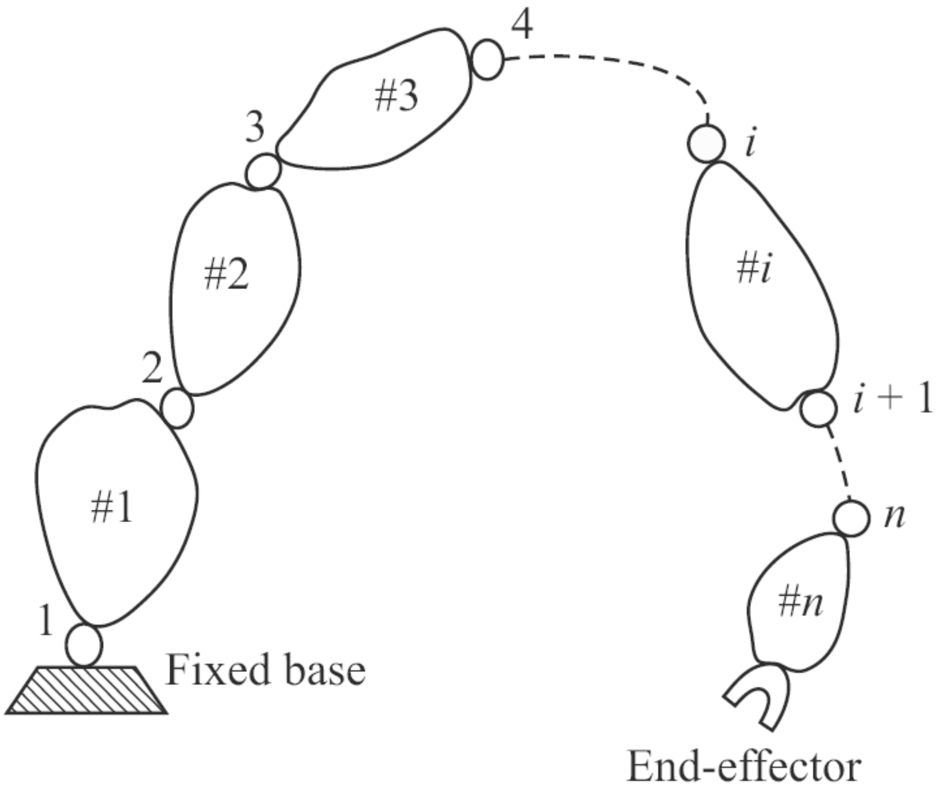
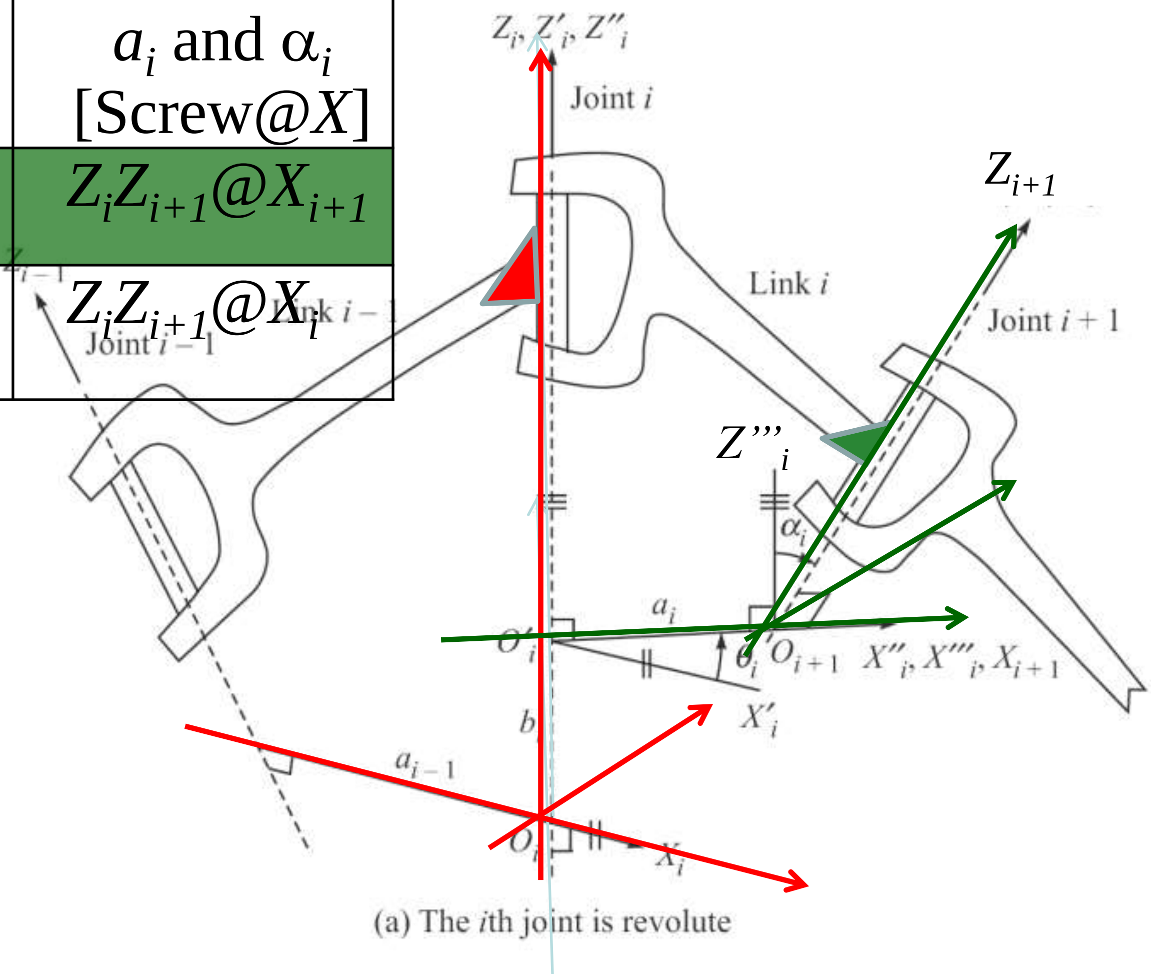


Fig. 5.27 Serial manipulator

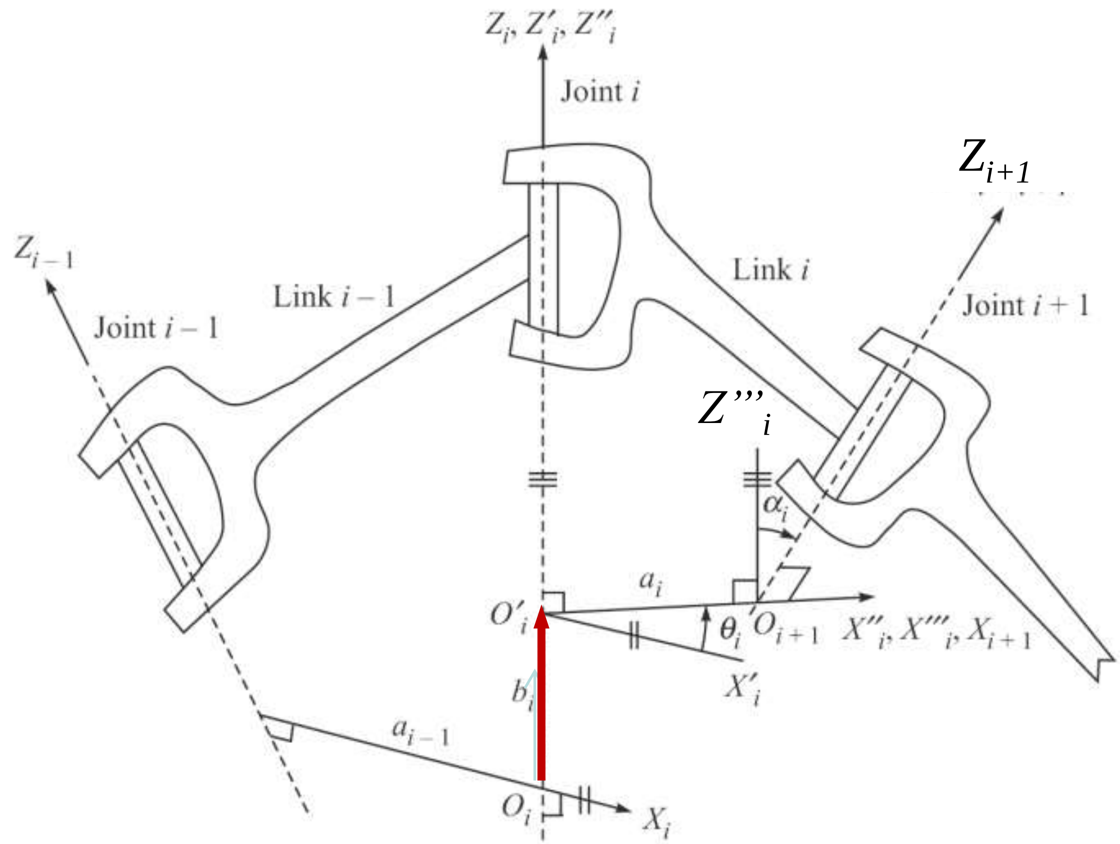
- Joint axis *i*: Link *i-1* + link *i*
- Link *i*: Fixed to **frame *i+1*** (Saha) / **frame *i*** (Craig)

DH	Variables b_i and θ_i [Screw@Z]	Constants a_i and α_i [Screw@X]
Saha	$X_iX_{i+1}@Z_i$	$Z_iZ_{i+1}@X_{i+1}$
Craig	$X_{i-1}X_i@Z_i$	$Z_iZ_{i+1}@X_i$



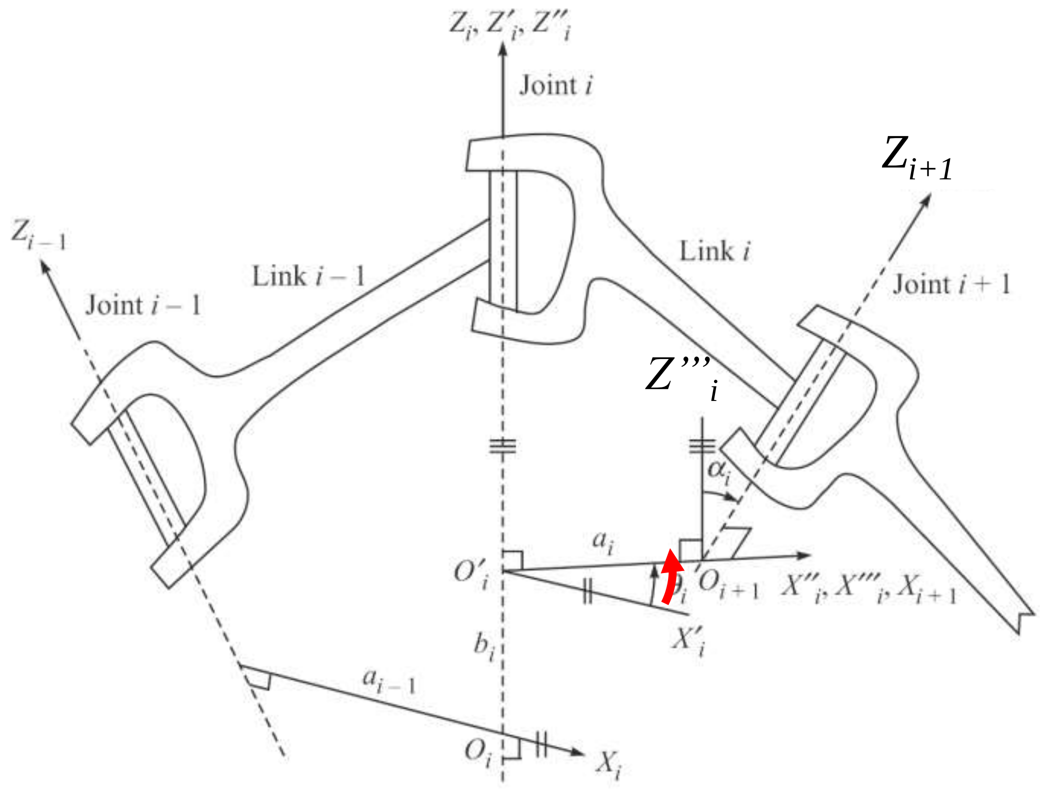
(a) The *i*th joint is revolute

- b_i (Joint offset): Length of the intersections of the common normals on the joint axis Z_i, i.e., O_i and O'_i. It is the relative position of links $i - 1$ and i. This is measured as the distance between X_i and X_{i+1} along



(a) The ith joint is revolute

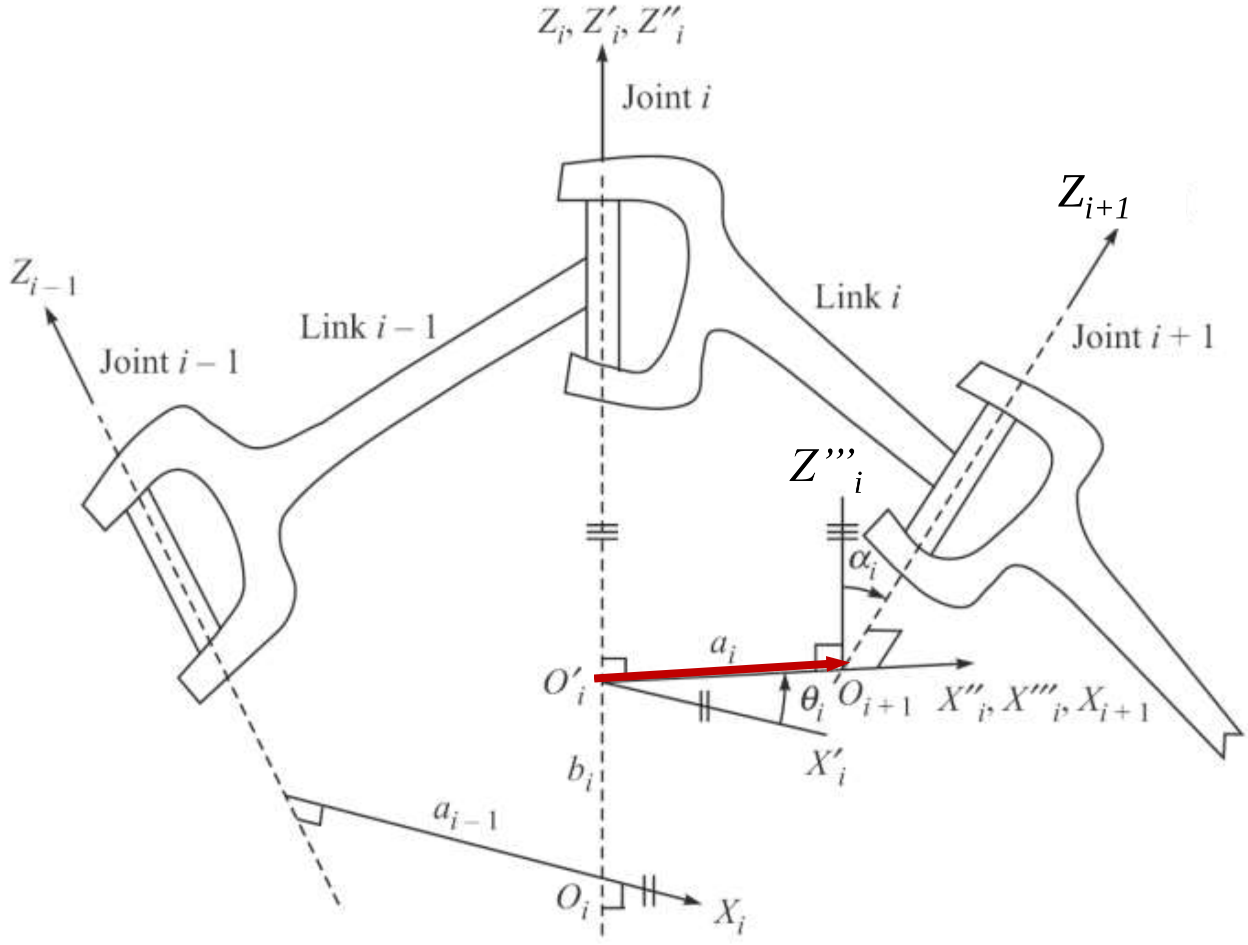
- θ_i (Joint angle): Angle between the orthogonal projections of the common normals, X_i and X_{i+1}, to a plane normal to the joint axes Z_i. Rotation is positive when it is made counter clockwise. It is the relative angle between links $i - 1$ and i. This is measured as the angle between X_i and X_{i+1} about Z_i.



(a) The ith joint is revolute

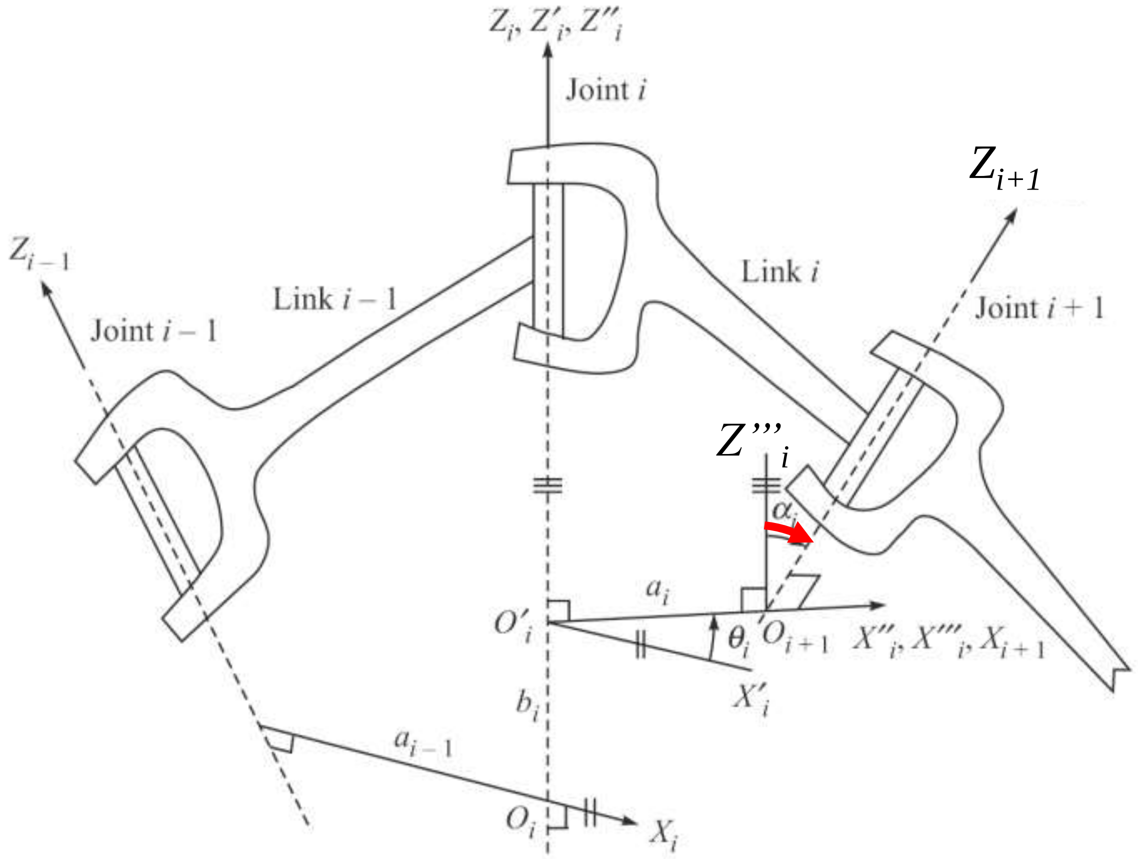
(a) The ith joint is revolute

- a_i (Link length): Length between the O'_i and O_{i+1}. This is measured as the distance between the common normals to axes Z_i and Z_{i+1} along X_{i+1}.



(a) The ith joint is revolute

- α_i (Twist angle): Angle between the orthogonal projections of joint axes, Z_i and Z_{i+1} onto a plane normal to the common normal. This is measured as the angle between the axes, Z_i and Z_{i+1}, about axis X_{i+1} to be taken positive when rotation is made counter clockwise.



(a) The ith joint is revolute

Revolute Joint

- DH@Z (Variable)
  - Joint offset (b)
  - Joint angle (θ)

- DH@X (Const.)
  - Link length (a)
  - Twist angle (α)

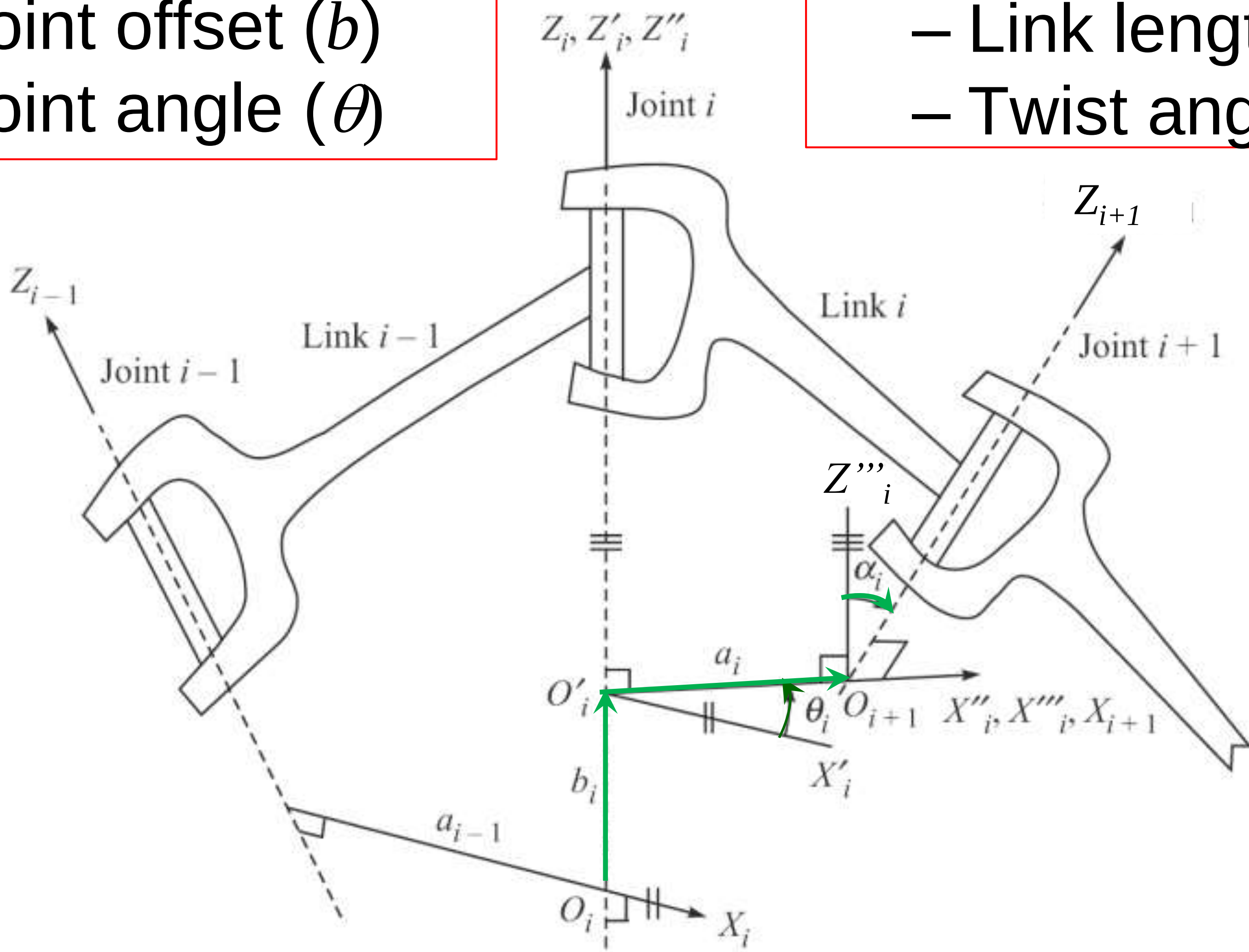


Fig. 5.28 (a) The ith joint is revolute

Mathematically

- Translation along Z_i

$$\mathbf{T}_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.60a)$$

- Rotation about Z_i

$$\mathbf{T}_\theta = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.60b)$$

- Translation along X_{i+1}

$$\mathbf{T}_a = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.60c)$$

- Rotation about X_{i+1}

$$\mathbf{T}_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.60d)$$

- Total transformation from Frame *i* to Frame *i+1*

$$\mathbf{T}_i = \mathbf{T}_b \mathbf{T}_\theta \mathbf{T}_a \mathbf{T}_\alpha \qquad \ldots (5.61a)$$

Do it yourself!

Rotation Matrix

Position

$$\mathbf{T}_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & \\ 0 & S\alpha_i & C\alpha_i & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\ldots (5.61b)$$

Spherical-type Arm

- DH-parameters

Link	b_i	θ_i	a_i	α_i
1				
2				
3				

Fill-up the DH parameters

RoboAnalyzer

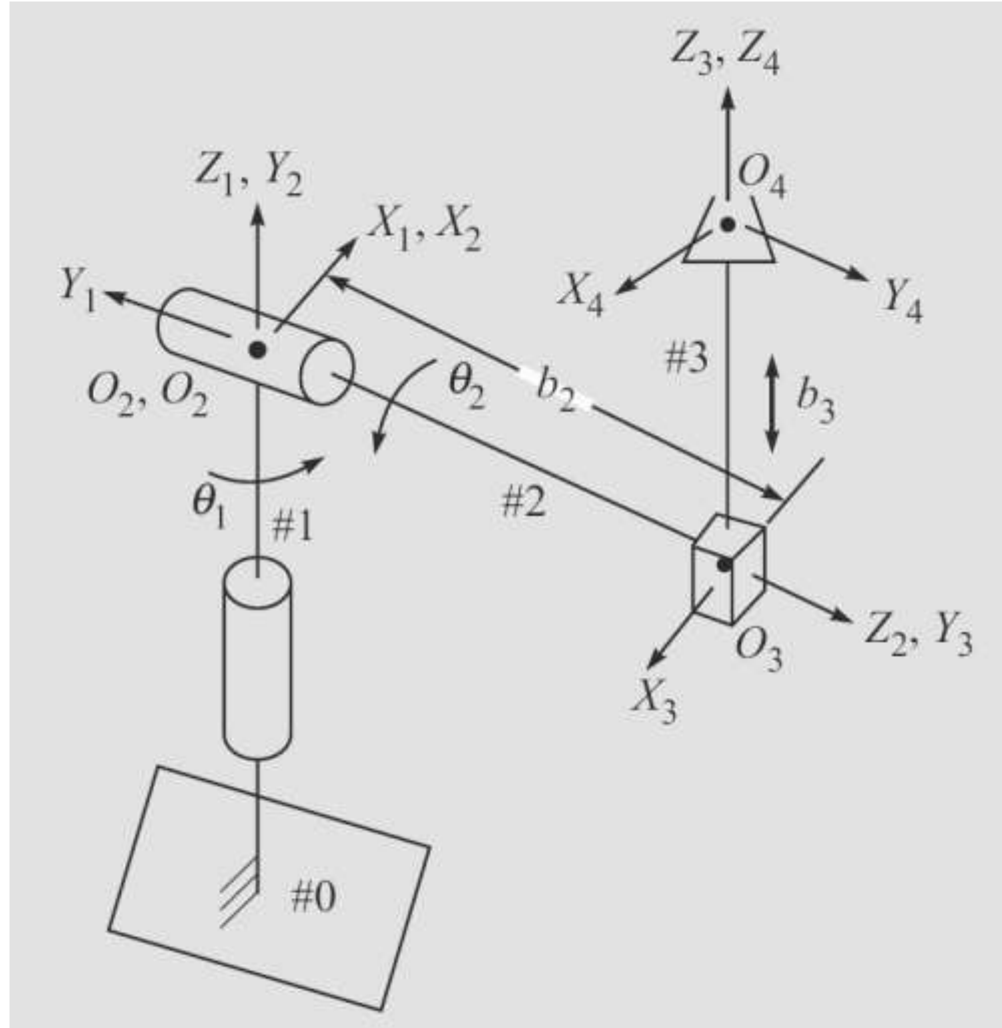


Fig. 5.32 A spherical arm

PUMA 560

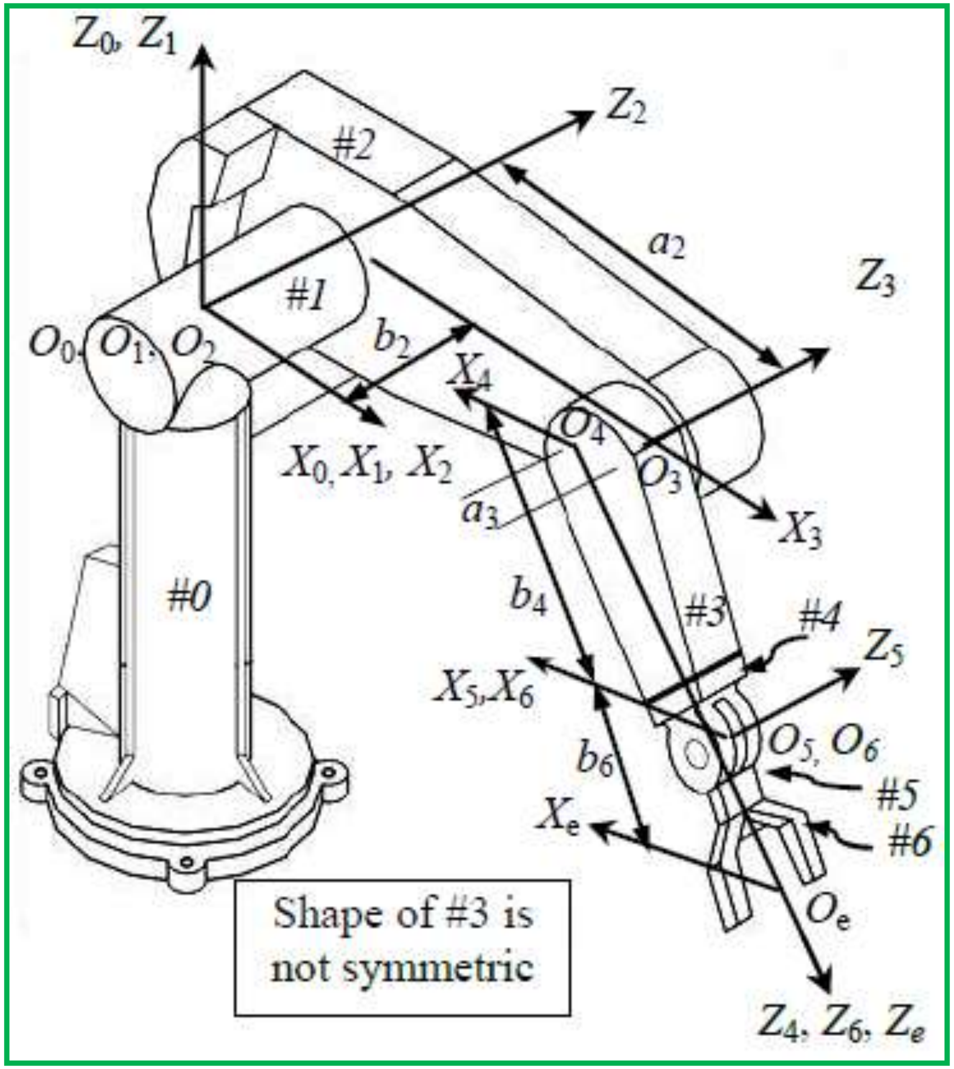


Fig. 5.35 PUMA 560 and its frames

i	Variable DH		Constant DH	
	b_i	θ_i	a_i	α_i
1	0	θ_1	0	$-\pi/2$
2	0	θ_2	a_2	0
3	B_3	θ_3	a_3	$-\pi/2$
4	b_4	θ_4	0	$\pi/2$
5	0	θ_5	0	$-\pi/2$
6	0	θ_6	0	0

Mc Graw Hill Education

Lecture 4 (SIT Sem. Rm.)

Forward and Inverse Kinematics (Ch. 6)

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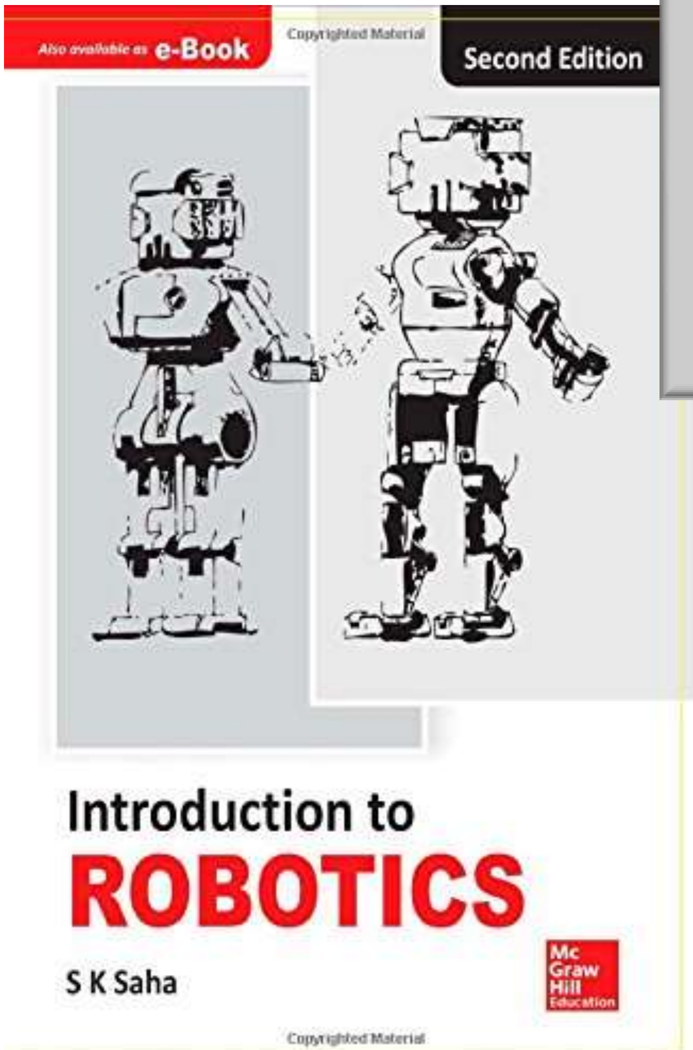


6

Kinematics

Forward and Inverse Kinematics

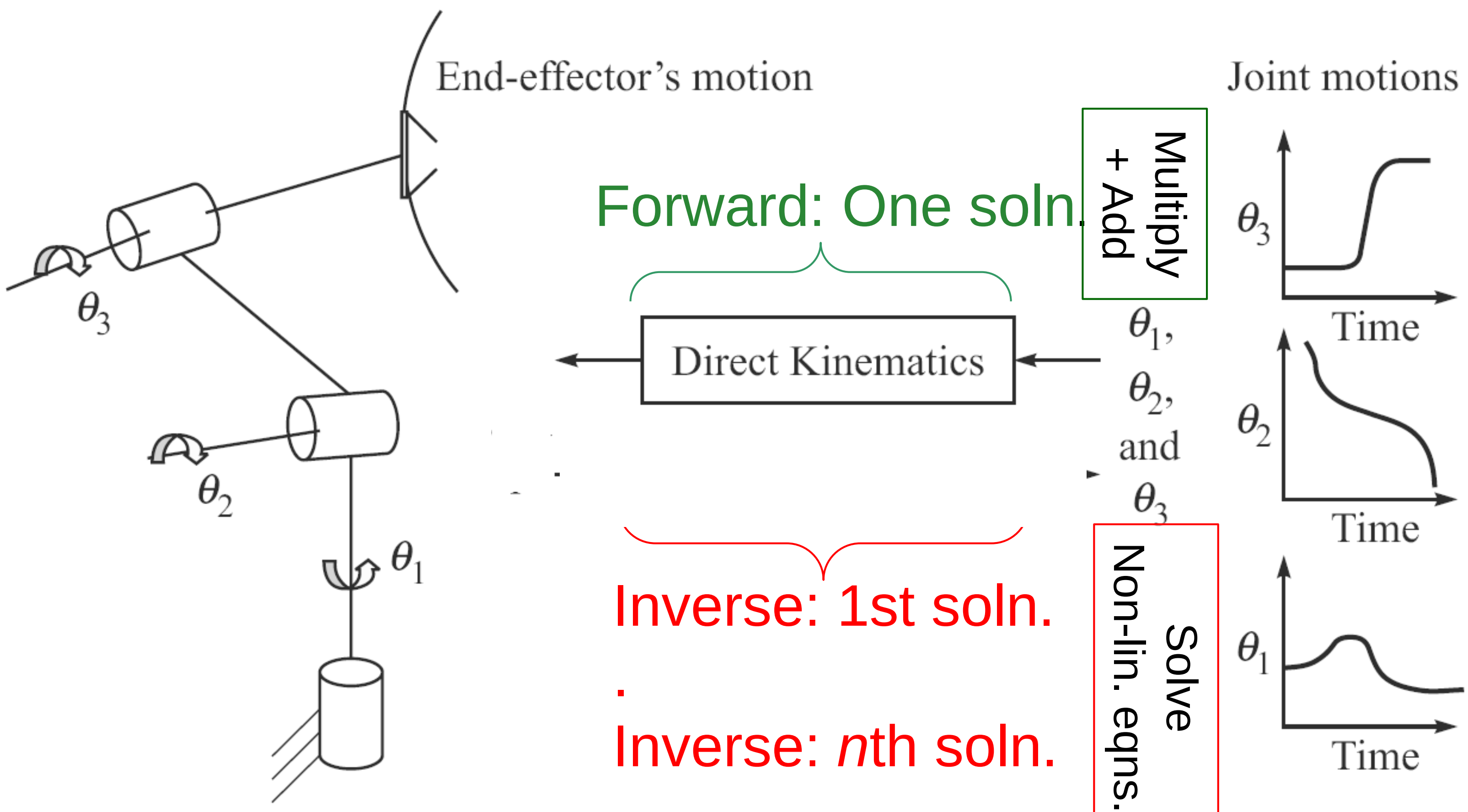


Fig. 6.1 Forward and inverse kinematics

Three-link Planar Arm

- DH-parameters

Link	b_i	θ_i	a_i	α_i
1	Fill-up the DH parameters			
2				
3				

- Frame transformations (Homogeneous)

$$\mathbf{T}_i = \begin{bmatrix} \text{Fill-up with the elements} & & & \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ for } i\text{=1,2,3}$$

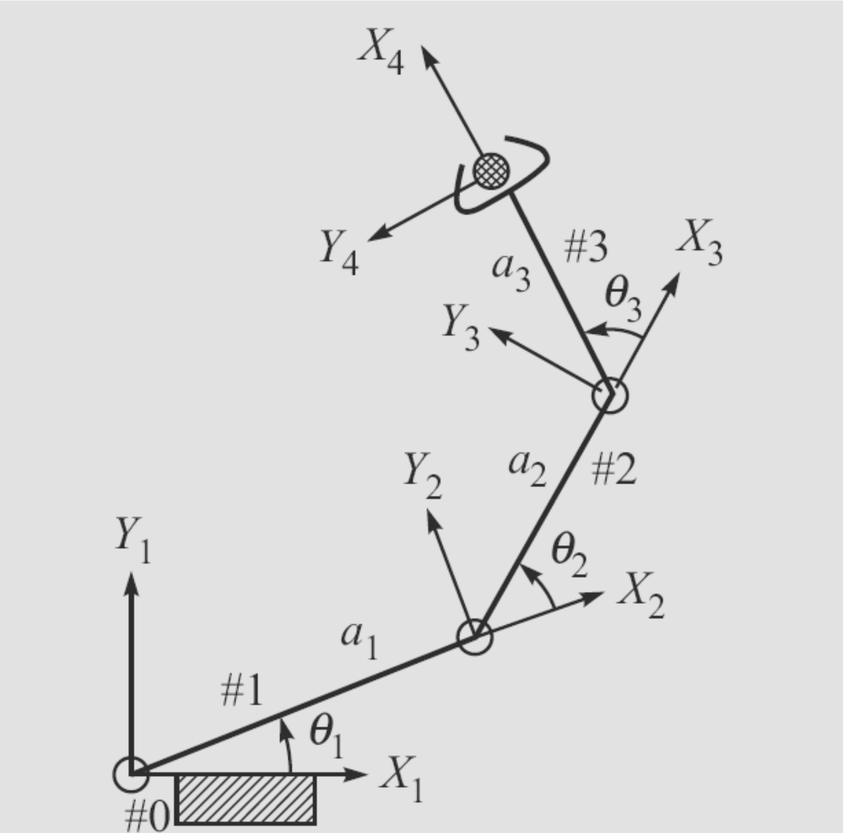


Fig. 5.29 A three-link planar arm

DH Parameters of Articulated Arm

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	0	$-\pi/2$
2	0	θ_2 (JV)	a_2	0
3	0	θ_3 (JV)	a_3	0

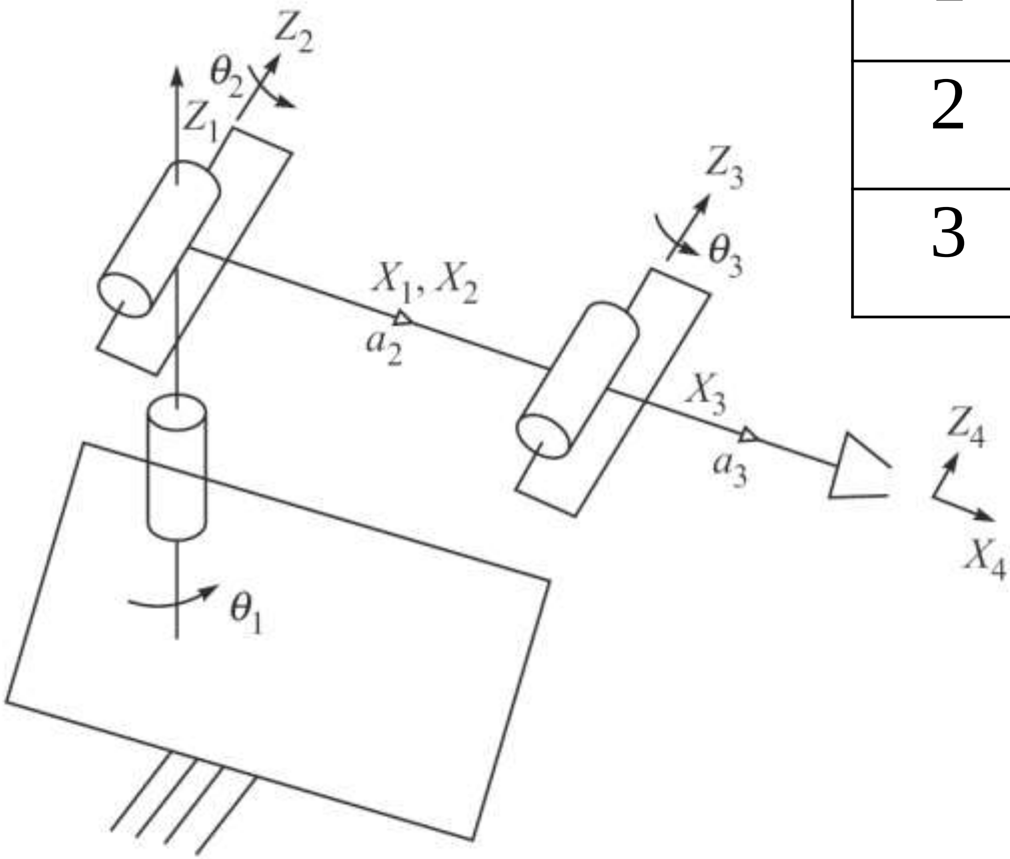


Fig. 5.29 An articulated arm

Matrices for Articulated Arm

$$\mathbf{T}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T}_2 \equiv \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_3 \equiv \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

MATLAB

$$\mathbf{T} \equiv \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 & c_1(a_2c_2 + a_3c_{23}) \\ s_1c_{23} & -s_1s_{23} & c_1 & s_1(a_2c_2 + a_3c_{23}) \\ -s_{23} & -c_{23} & 0 & -(a_2s_2 + a_3s_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (6.11)$$

Inverse Kinematics

- Unlike Forward Kinematics, general solutions are not possible.
- Several architectures are to be solved differently.

Two-link Arm

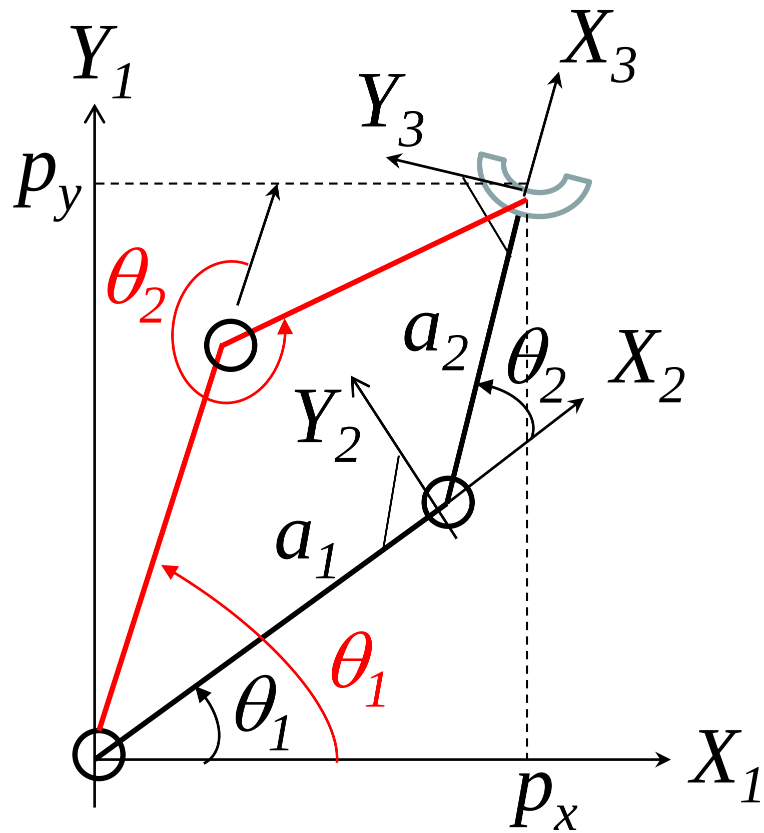
$$p_x = a_1 c_1 + a_2\, c_{12}$$

$$p_y = a_1 s_1 + a_2\, s_{12}$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2\, a_1\, a_2}$$

$$s_2 = \pm\sqrt{1 - c_2^2}$$

$$\theta_2 = \text{atan2}\,(s_2, c_2)$$



RoboAnalyzer

$$s_1 = \frac{(a_1 + a_2 c_2)\, p_y - a_2 s_2 p_x}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2a_1 a_2 c_2 = p_x^2 + p_y^2$$

$$c_1 = \frac{(a_1 + a_2 c_2)\, p_x + a_2 s_2 p_y}{\Delta}$$

$$\theta_1 = \text{atan2}\,(s_1, c_1)$$

Inverse Kinematics of 3-DOF RRR Arm

$$\varphi = \theta_1 + \theta_2 + \theta_3 \quad \ldots (6.18a)$$

$$p_x = a_1 c_1 + a_2 c_{12} + a_3 c_{123} \quad \ldots (6.18b)$$

$$p_y = a_1 s_1 + a_2 s_{12} + a_3 s_{123} \quad \ldots (6.18c)$$

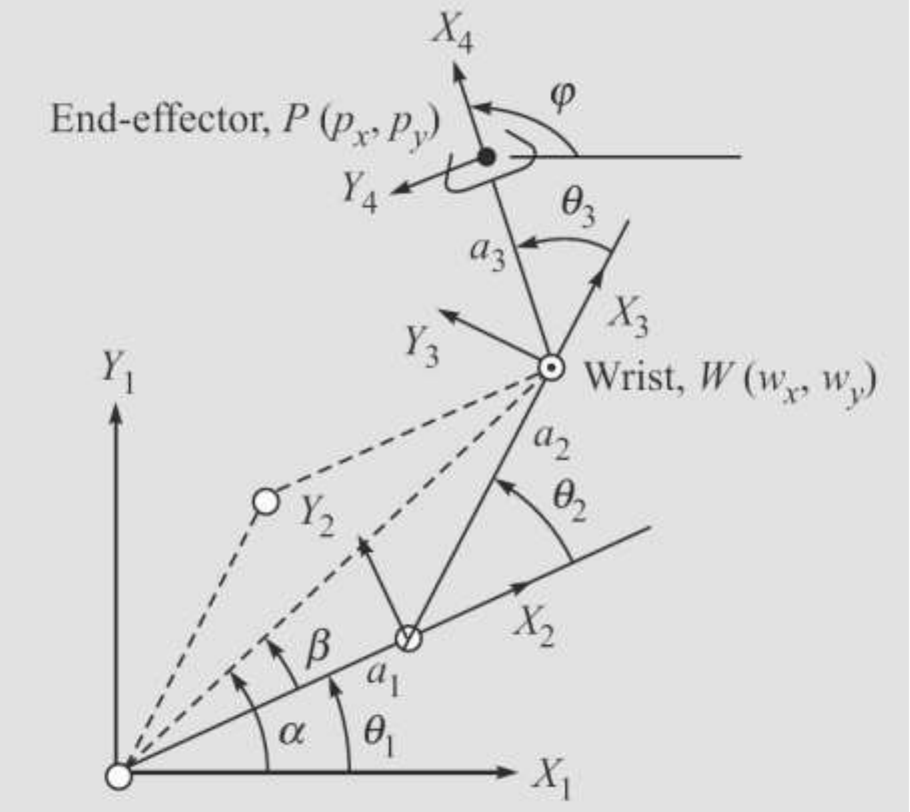


Fig. 6.3 Kinematics of a three-link planar arm

$$w_x = p_x - a_3 c\,\varphi = a_1 c_1 + a_2 c_{12} \quad \ldots (6.19a)$$

$$w_y = p_y - a_3 s\,\varphi = a_1 s_1 + a_2 s_{12} \quad \ldots (6.19b)$$

$$w_x^2 + w_y^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 c_2 \qquad \ldots (6.20a)$$

$$c_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2\, a_1\, a_2} \qquad s_2 = \pm\sqrt{1 - c_2^2} \qquad \ldots (6.20b,c)$$

$$\theta_2 = \text{atan2}\,(s_2, c_2) \qquad \ldots (6.21)$$

$$w_x = (a_1 + a_2 c_2) c_1 - a_2 s_1 s_2 \qquad \ldots (6.22a)$$

$$w_y = (a_1 + a_2 c_2) s_1 + a_2 c_1 s_2 \qquad \ldots (6.22b)$$

$$s_1 = \frac{(a_1 + a_2 c_2) w_y - a_2 s_2 w_x}{\Delta} \qquad c_1 = \frac{(a_1 + a_2 c_2) w_x + a_2 s_2 w_y}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = w_x^2 + w_y^2 \qquad \ldots (6.23a,b)$$

$$\theta_1 = \text{atan2}\,(s_1, c_1) \qquad \ldots (6.23c)$$

$$\theta_3 = \varphi - \theta_1 - \theta_2 \qquad \ldots (6.24)$$

Numerical Example

- An RRR planar arm (Example 6.15). Input

$$\mathbf{T} \equiv \begin{bmatrix} \text{Rotation Matrix} & & & \text{Origin of end-effector frame} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} 4.23 \\ 1.86 \\ 0 \end{matrix}$$

where $\phi = 60^o$, and $a_1 = a_2 = 2$ units, and $a_3 = 1$ unit.

Do it yourself → Verify using RoboAnalyzer

Using eqs. (6.13b-c), $c_2 = 0.866$, and $s_2 = 0.5$,

$$\theta_2 = 30^o$$

Next, from eqs. (6.16a-b), $s_1 = 0$, and $c_1 = 0.866$.

$$\theta_1 = 0^o.$$

MATLAB program

Finally, from eq. (6.17) ,

$$\theta_3 = 30^o.$$

Therefore

$$\theta_1 = 0^o\ \theta_2 = 30^o, \text{ and } \theta_3 = 30 \qquad \ldots(6.30b)$$

The positive values of s_2 was used in evaluating $\theta_2 = 30^o$.

The use of negative value would result in :

$$\theta_1 = 30^o\ \theta_2 = -30^o, \text{ and } \theta_3 = 60^o \qquad \ldots(6.30c)$$

Watch

- Forward and Inverse Kinematics: Watch 3/3 of IGNOU Lectures [29min]

https://www.youtube.com/watch?v=duKD8cvtBTI

- For more clarity: Watch 12 of Addis Ababa Lectures [77 min]

[https://www.youtube.com/watch?v=NXWzk1toze4

- Robotics (13 of Addis Ababa Lectures): Inverse Kinematics [82 min]

https://www.youtube.com/watch?v=uIP3YiJLiEM

Velocity Analysis

Jacobian maps joint rates into end-effector's velocities. It depends on the manipulator configuration.

$$\text{twist of end-effector}: \mathbf{t}_e \equiv \begin{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{v}_e \end{bmatrix}; \text{ Joint rates}: \dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

$$\mathbf{t}_\mathbf{e} = \mathbf{J}\dot{\boldsymbol{\theta}} \quad \text{where } \mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_n \end{bmatrix} \text{ and}$$

$$\boxed{\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_\mathbf{n} \\ \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} & \cdots & \mathbf{e}_\mathrm{n} \times \mathbf{a}_{ne} \end{bmatrix}} \quad ..(6.86)$$

$$\mathbf{j}_\mathrm{i} \equiv \begin{bmatrix} \mathbf{e}_\mathrm{i} \\ \mathbf{e}_\mathrm{i} \times \mathbf{a}_\mathrm{ie} \end{bmatrix}, \text{ if Joint } i \text{ is revolute} \quad \mathbf{j}_i \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{e}_i \times \mathbf{a}_{ie} \end{bmatrix}, \text{ if Joint } i \text{ is prismatic}$$

Jacobian of a 2-link Planar Arm

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} \end{bmatrix}$$

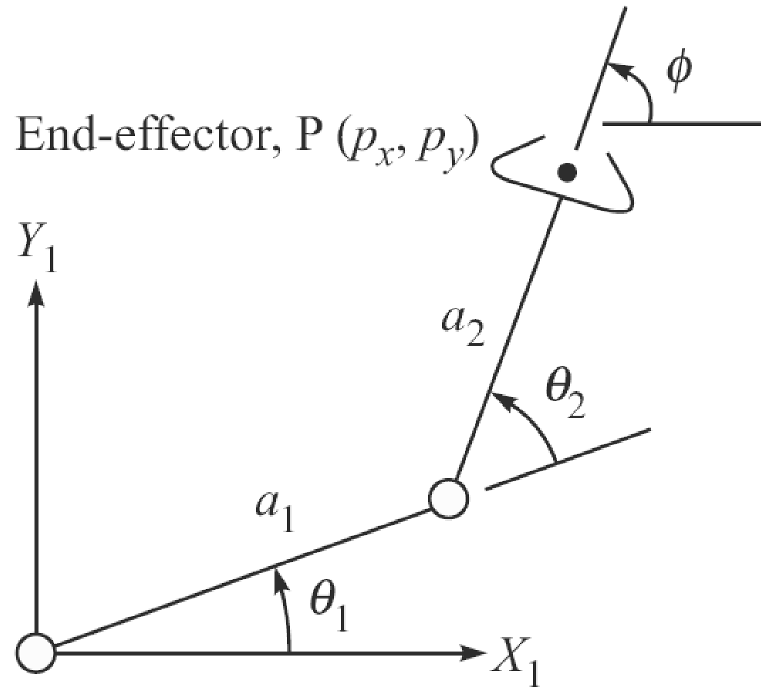
where $\mathbf{e}_1 \equiv \mathbf{e}_2 \equiv [0 \quad 0 \quad 1]^T$

$$\mathbf{a}_{1e} \equiv \mathbf{a}_1 + \mathbf{a}_2$$

$$\equiv [a_1 c_1 + a_2 c_{12} \quad a_1 s_1 + a_2 s_{12} \quad 0]^T$$

$$\mathbf{a}_{2e} \equiv \mathbf{a}_2$$

$$\equiv [a_2 c_{12} \quad a_2 s_{12} \quad 0]^T$$



$$\text{Hence, } \mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

Example: Singularity of 2-link RR Arm

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

$$\theta_2 = 0 \text{ or } \pi$$

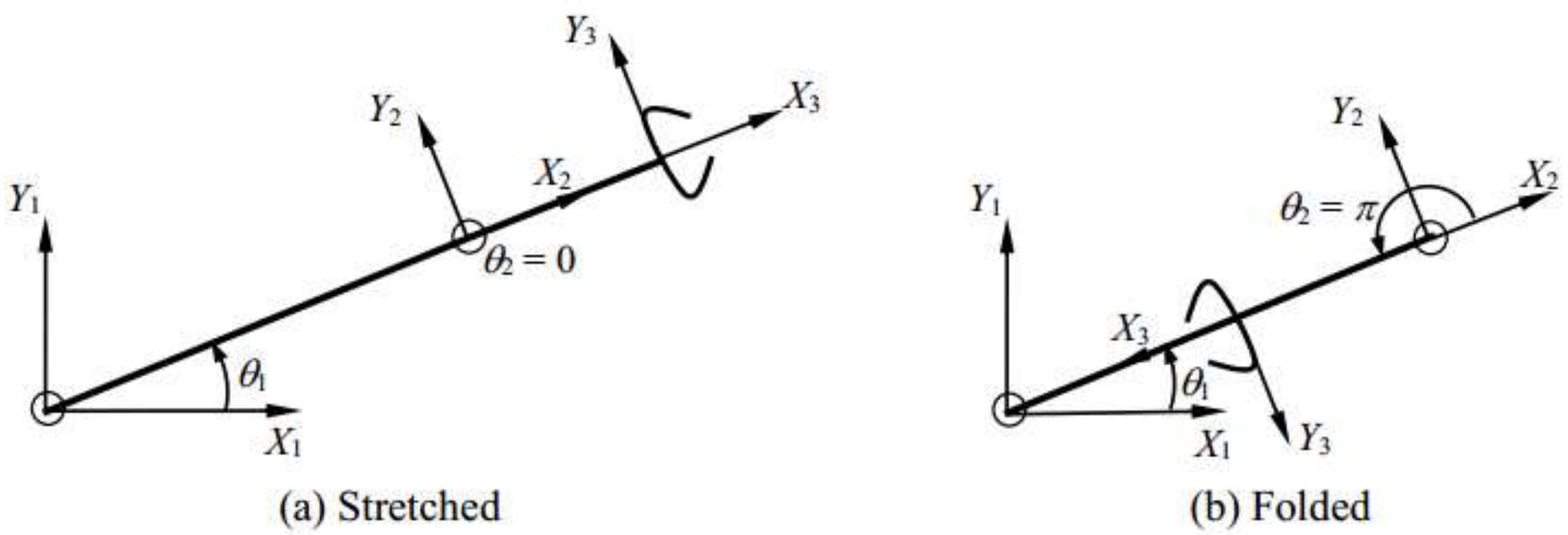


Figure 7.9 Singular configurations of a two-link planar arm

Lecture 4 (SIT Sem. Rm.)

Forward and Inverse Kinematics (Ch. 6)

by

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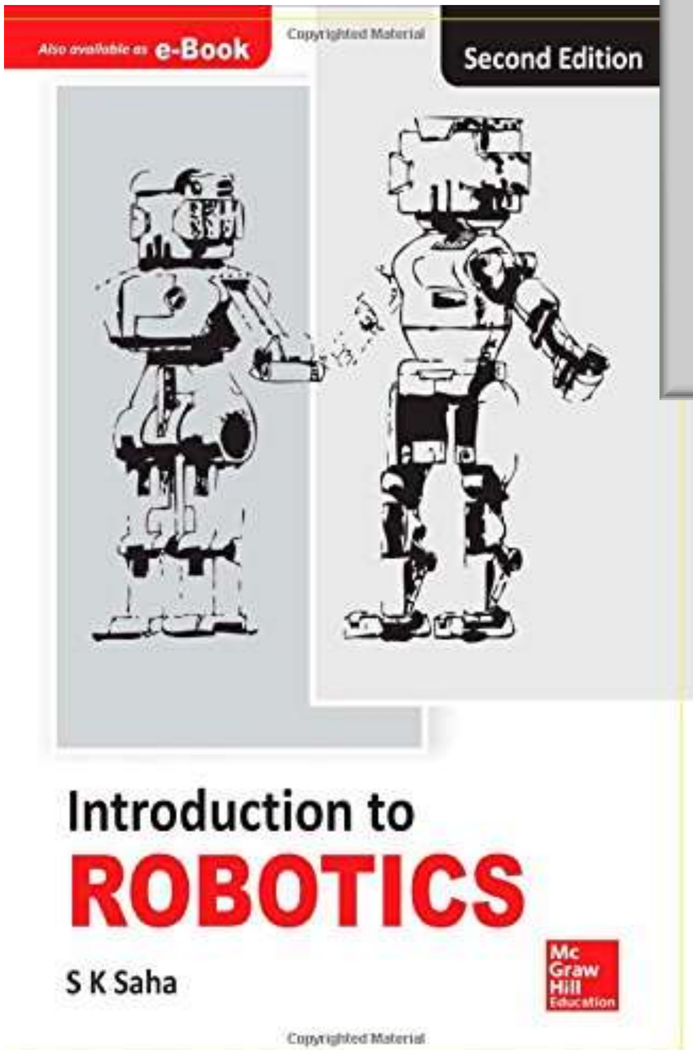
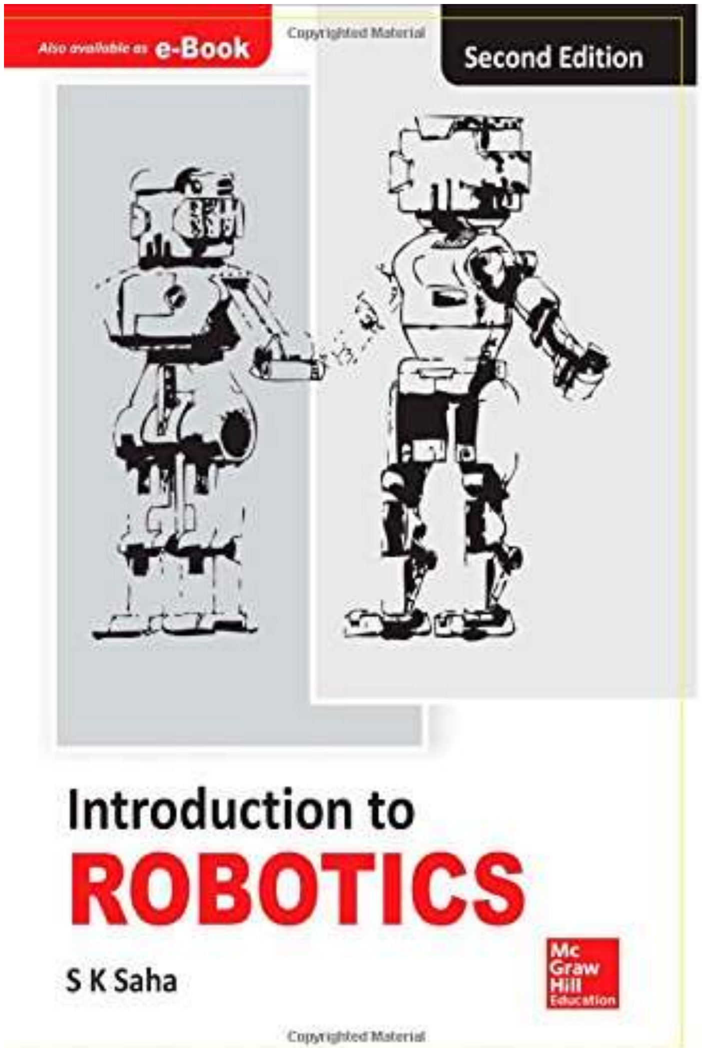
Aug. 12, 2015 (W)@JRL301(Robotics Tech.)





Statics and Manipulator Design (Ch. 7)

Principle of Virtual Work

$$\mathbf{w}_e^T \delta \mathbf{x} = \boldsymbol{\tau}^T \delta \boldsymbol{\theta} \quad \text{... (7.28)}$$

- Relation between two virtual displacements (Can be derived from velocity expression)

$$\delta \mathbf{x} = \mathbf{J} \delta \boldsymbol{\theta} \quad \text{... (7.29)}$$

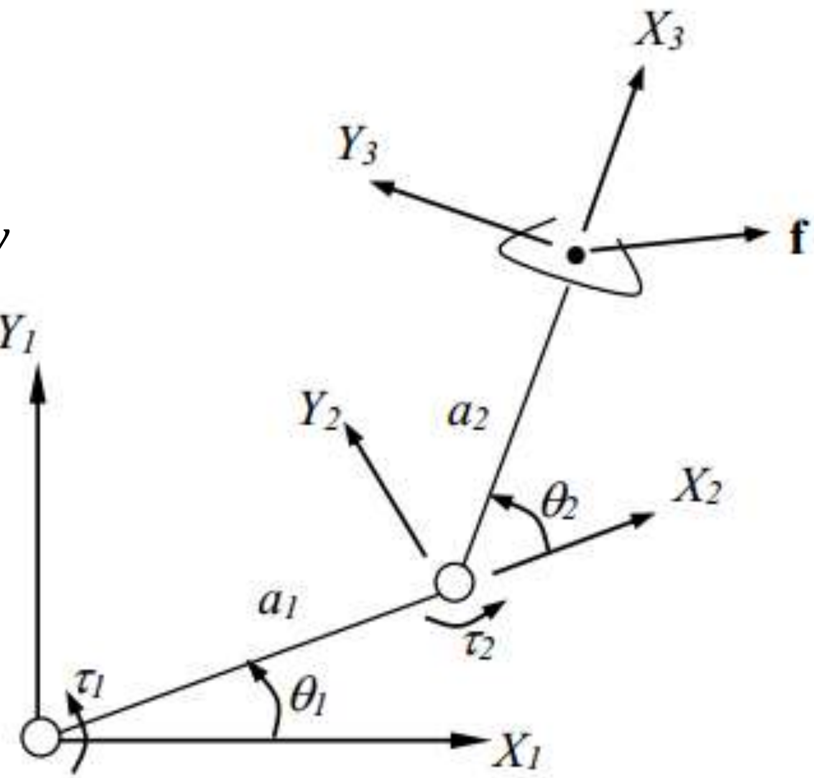
$$\mathbf{w}_e^T \mathbf{J} \delta \boldsymbol{\theta} = \boldsymbol{\tau}^T \delta \boldsymbol{\theta} \Rightarrow \mathbf{w}_e^T \mathbf{J} = \boldsymbol{\tau}^T \quad \text{... (7.31)}$$

$$\boxed{\boldsymbol{\tau} = \mathbf{J}^T \mathbf{w}_e} \quad \text{... (7.32)}$$

Example: 2-link RR Planar Arm

$$\tau_1 = [\mathbf{e}_1]_1^T [\mathbf{n}_{01}]_1$$

$$= a_1 f_x s\theta_2 + (a_2 + a_1 c\theta_2) f_y$$



$$\tau_2 = [\mathbf{e}_2]_2^T [\mathbf{n}_{12}]_2 = a_2 f_y$$

$$\boxed{\boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}} \mathbf{f}}$$

$$\boldsymbol{\tau} \equiv \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \mathbf{J}^T \equiv \begin{bmatrix} a_1 s\theta_2 & a_1 c\theta_2 + a_2 & 0 \\ 0 & a_2 & 0 \end{bmatrix} \quad \mathbf{f} \equiv \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

Two Jacobian Matrices

- From Statics

$$\mathbf{J} \equiv \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

- From Kinematics

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

Jacobian from Statics in Frame 1

$$[\mathbf{J}]_1 \equiv \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \\ 0 & 0 \end{bmatrix} \quad \ldots (7.34)$$

- Without the last row, it is the same as the one from kinematics ← Should be!

Manipulator Design

- High investment in robot usage → low technological level of mechanical structure
- Functional Requirements
- Kinetostatic Measures
- Structural Design and Dynamics
- Economics

Functional Requirements of a Robot

- Payload
- Mobility
- Configuration

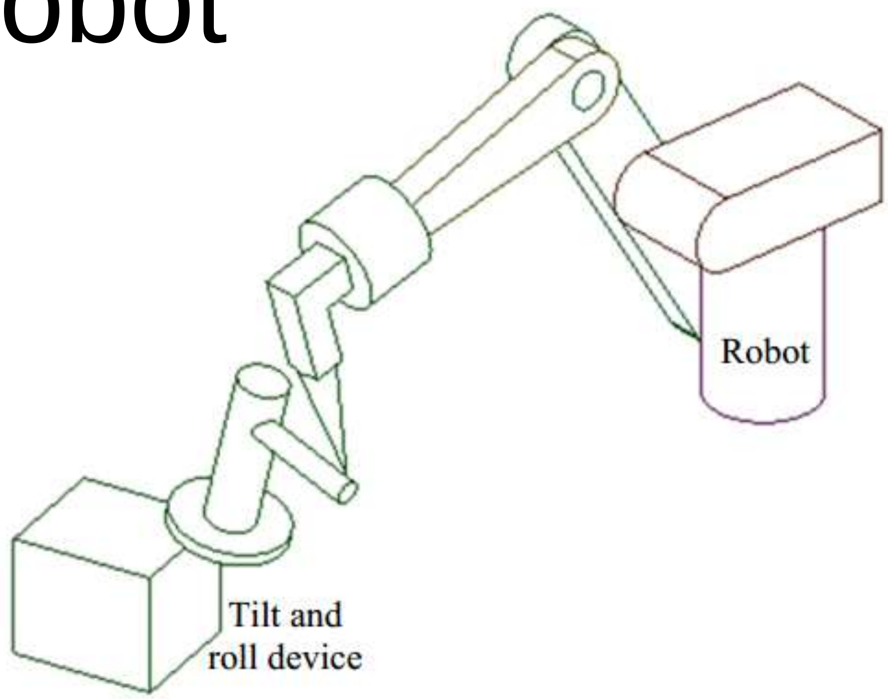


Figure 7.6 A tilt and roll device provides additional DOF to the robot system

- Speed, Accuracy and Repeatability
- Actuators and Sensors

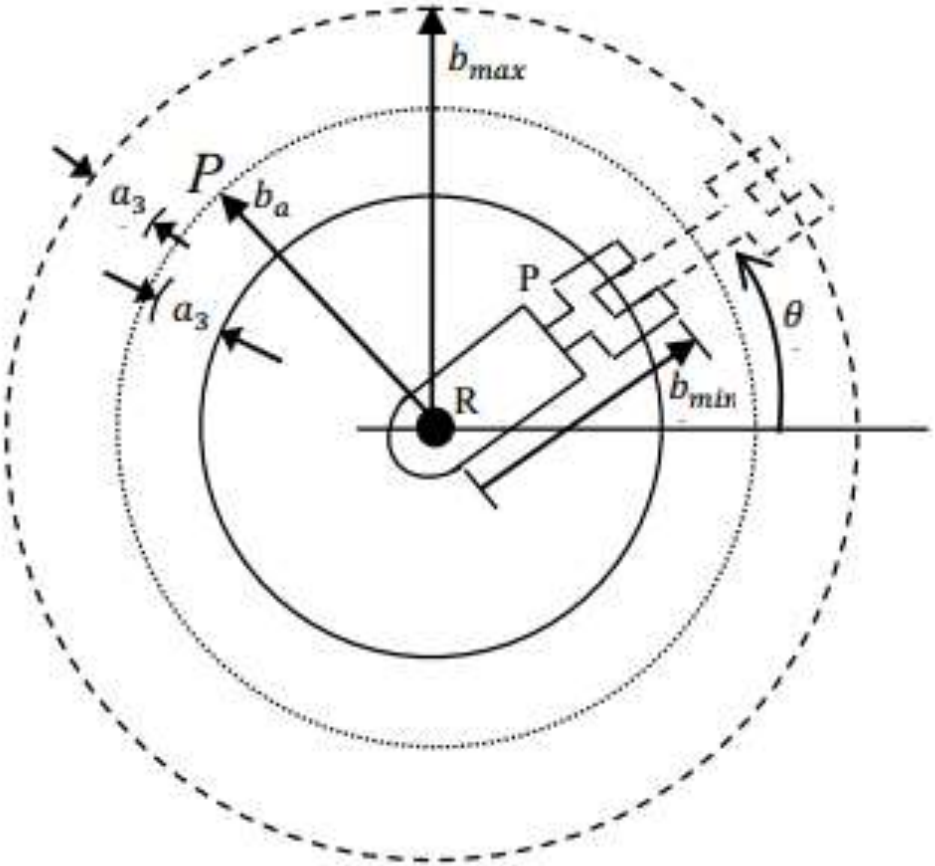


Figure 7.7 Workspace of a 2-DOF RP planar manipulator

$$b_{\min} \le b \le b_{\max}, \text{ for } 0^o \le \theta \le 360^o$$

Dexterity and Manipulability

- Dexterity → $w_d = \det(\mathbf{J})$ … (7.44)
- Manipulability → $w_m = \sqrt{\det(\mathbf{J}\mathbf{J}^T)}$
- Nonredundant manipulator → square Jacobian

$$w_m = \left|\det(\mathbf{J})\right| \qquad w_d = w_m$$

Motor Selection (Thumb Rule)

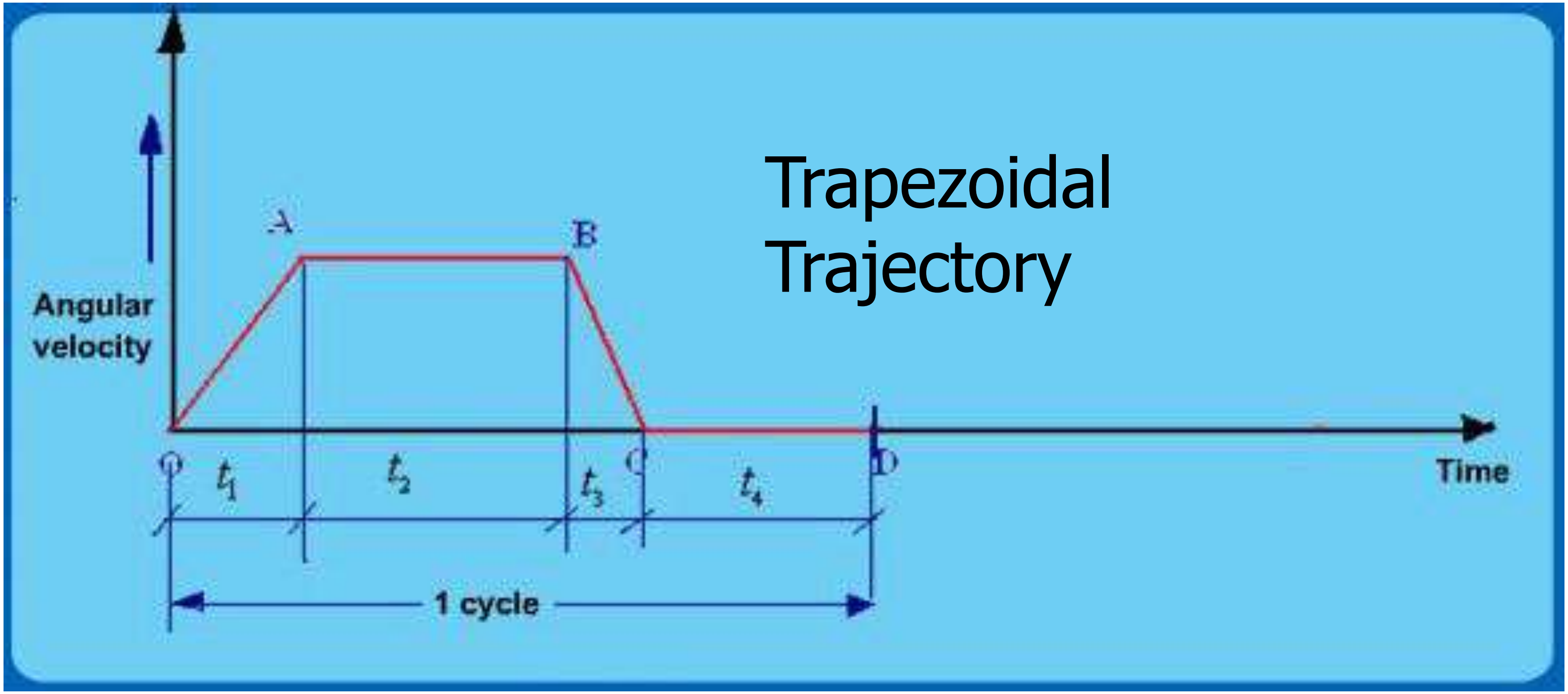
- Rapid movement with high torques (> 3.5 kW): Hydraulic actuator
- < 1.5 kW (no fire hazard): Electric motors
- 1-5 kW: Availability or cost will determine the choice

Simple Calculation

2 m robot arm to lift 25 kg mass at 10 rpm

- Force = 25 x 9.81 = 245.25 N
- Torque = 245.25 x 2 = 490.5 Nm
- Speed = 2π x 10/60 = 1.047 rad/sec
- Power = Torque x Speed = 0.513 kW
- Simple but sufficient for approximation

Practical Application



Subscript *l* for load; *m* for motor;

$G = \omega_l/\omega_m$ (< 1); η: Motor + Gear box efficiency

Accelerations & Torques

Ang. accn. during t_1: $$\alpha_1 = \frac{\omega_2 - 0}{t_1}$$

Ang. accn. during t_2: Zero (Const. Vel.)

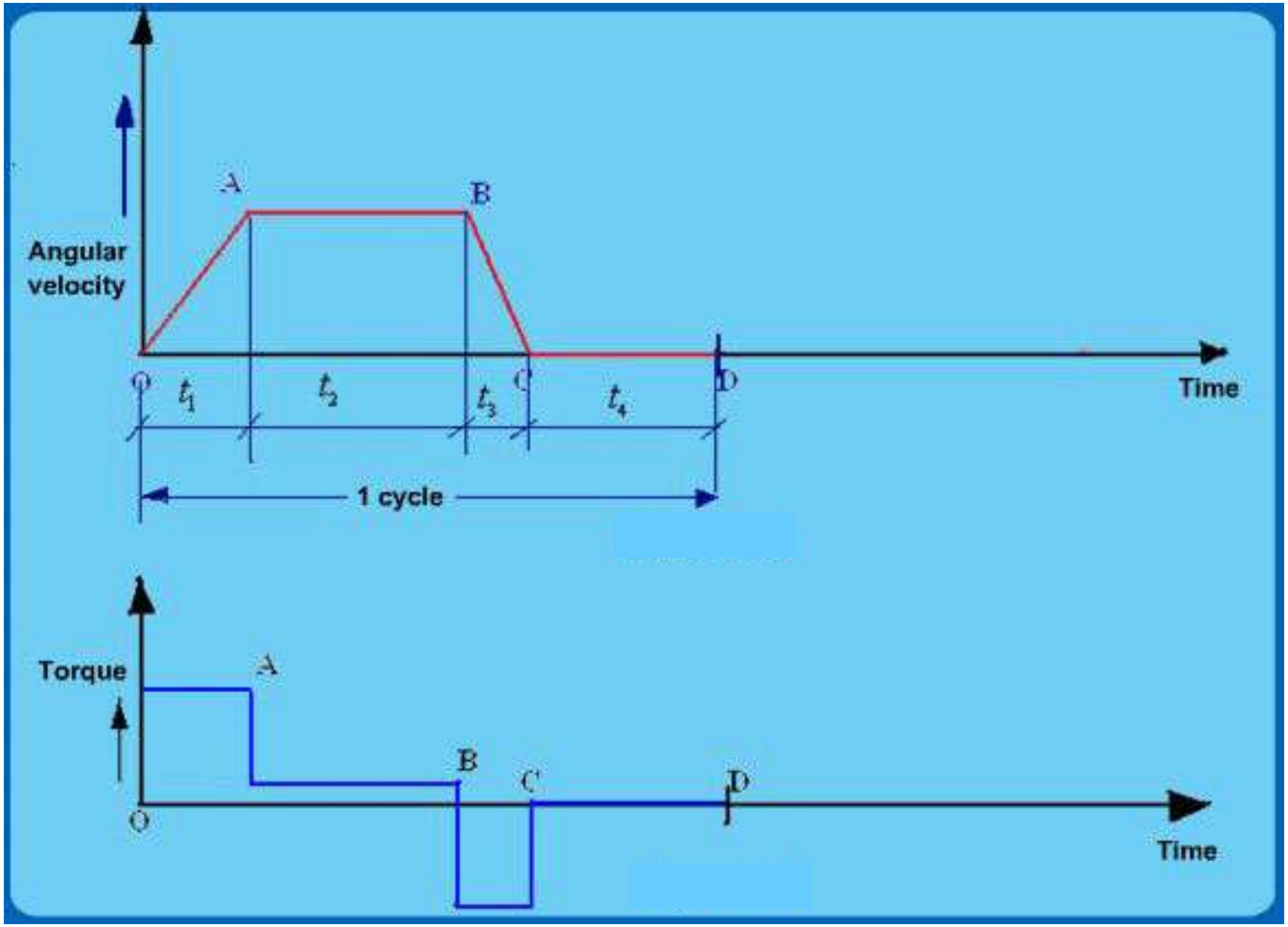
Ang. accn. during t_3: $$\alpha_3 = \frac{\omega_b - 0}{t_3}$$

Torque during t_1: $T_1 =$ $$\left(I_m + \frac{G^2}{\eta} I_l\right)\alpha_1 + T_f \frac{G}{\eta}$$

Torque during t_2: $T_2 =$ $$T_f \frac{G}{\eta}$$

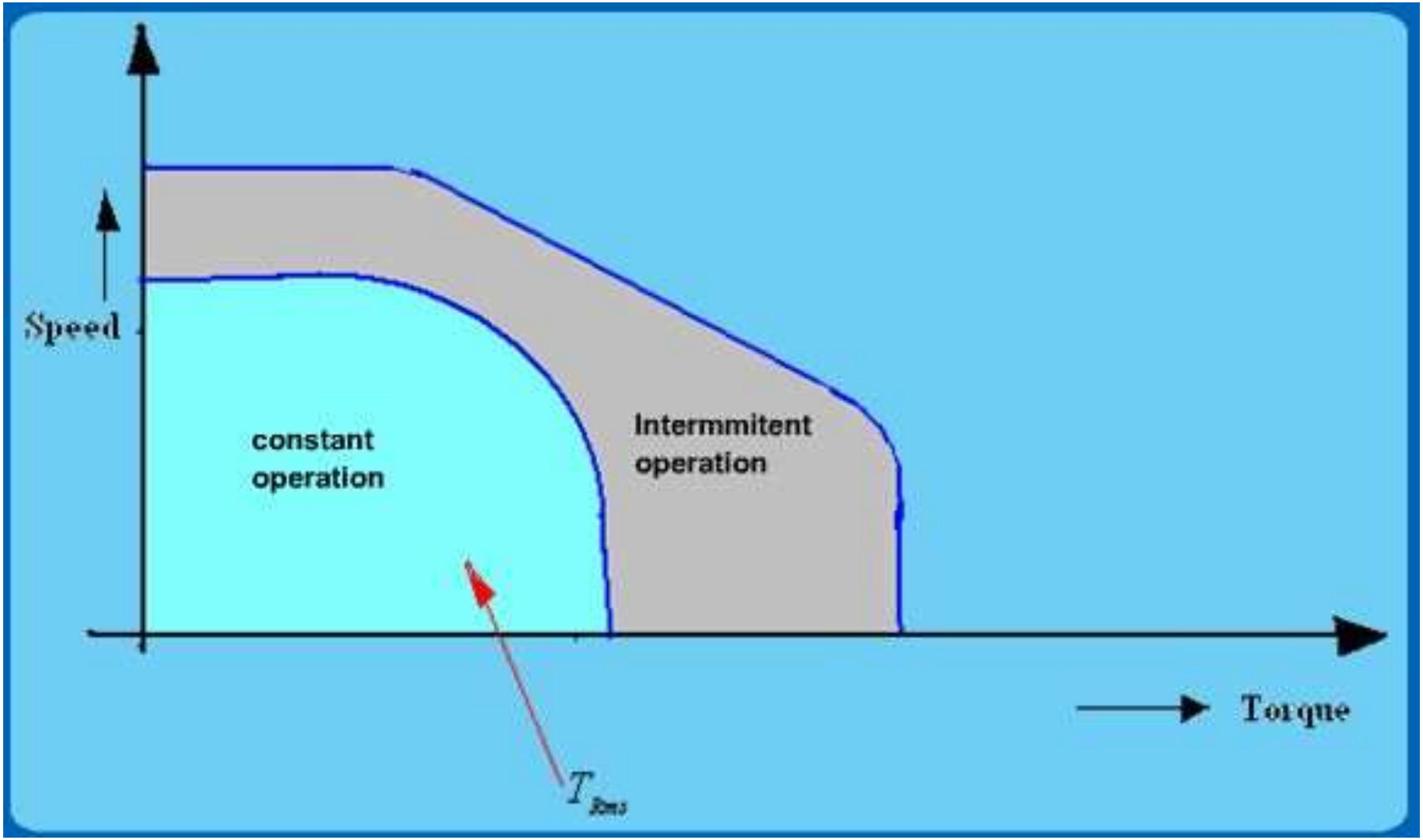
Torque during t_3: $T_3 =$ $$\left(I_m + \frac{G^2}{\eta} I_l\right)\alpha_3 - T_f \frac{G}{\eta}$$

RMS Value



$$T_{RMS} = \sqrt{\frac{(T_1^2 \times t_1) + (T_2^2 \times t_2) + (T_3^2 \times t_3) + (zero)t_4}{t_1 + t_2 + t_3 + t_4}}$$

Motor Performance



Final Selection

- Peak speed and peak torque requirements , where T_{Peak} is max of (magnitudes) T_1, T_2, and T_3
- Use individual torque and RMS values + Performance curves provided by the manufacturer.
- Check heat generation + natural frequency of the drive.

Dynamics and Control Measures

- Rule of Thumb

$$\omega_n \leq \frac{1}{2}\omega_r \qquad \ldots (7.51)$$

ω_n : closed-loop natural frequency

ω_r : lowest structural resonant frequency

Manipulator Stiffness

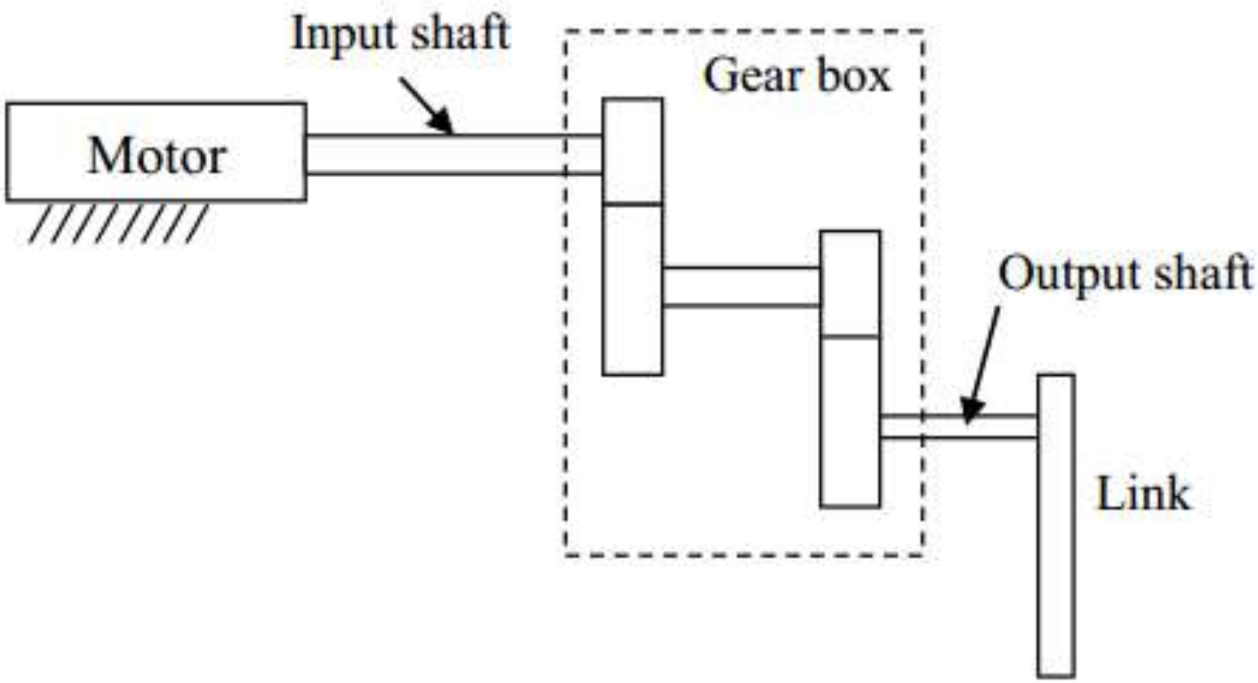


Figure 7.11 Shaft assembly of a link

$$\frac{1}{k_e} = \frac{1}{\eta^2 k_1} + \frac{1}{k_2} \quad \ldots (7.48)$$

$k_e \equiv$ equivalent stiffness

$\eta \equiv$ gear ratio

Link Material Selection

- Mild (low carbon) steel:

  S_y = 350 Mpa; S_u = 420 Mpa
- High alloyed steel

  S_y = 1750-1900 Mpa; S_u = 2000-2300 Mpa
- Aluminum
- S_y = 150-500 Mpa; S_u = 165-580 Mpa

Driver Selection

- Driver of a DC motor: A hardware unit which generates the necessary current to energize the windings of the motor
- Commercial motors come with matching drive systems

Summary

- Forward Kinematics
- Inverse kinematics
  - A spatial 6-DOF wrist-portioned has 8 solutions
- Velocity and Jacobian
- Mechanical Design

THANK YOU

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