ROBOT SELECTION USING DeNOC-BASED DYNAMICS

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Abstract - Robot selection for an application is generally done based on experience, intuition and at most using the kinematic considerations like workspace, manipulability, etc. Dynamics is ignored at this stage even though it is widely used for robot control and simulation purposes. This paper uses dynamic characteristics of a robot, namely, the computational simplicity of the matrices associated with its dynamic equations of motion, i.e., the Generalized Inertia Matrix, (GIM) and the Matrix of Convective Inertia (MCI) terms. Simplification of these matrices occurs due to specific values of the robot parameters like link masses and lengths, joint sequences, etc. Hence, these parameters can be used as design variables to make some elements of the GIM and MCI vanish or constant. Explicit expressions of the GIM and MCI elements are available due to the use of the concept of the Decoupled Natural Orthogonal Compliment (DeNOC) matrices, introduced elsewhere, in deriving the dynamic equations of motion of the robots under study. Selection of robot parameters based on GIM and MCI simplification eases both the control and simulation tasks of the chosen robot, hence, will improve its speed, precision and stability.

Keywords: Robot, Selection, DeNOC, GIM, MCI, Computational Complexity.

1 INTRODUCTION

A robot is characterized by its degree of freedom, number of joints, type of joints, joint placement, link lengths and shapes, and their orientation which influence its performances, namely, the workspace, manipulability, ease and speed of operation, etc. The speed of operation significantly depends on the complexities of the kinematic and dynamic equations and their computations. Hence, in order to select a suitable robot, both aspects of kinematics and dynamics should be looked into. Generally, kinematic characteristics like workspace, etc. are considered for the selection of a robot in an application. Dynamics is neglected even though they are widely used for control and simulation of robots. In this paper, simplicity of dynamic model and its computations are emphasized, particularly, with respect to the Generalized Inertia Matrix (GIM) and the Matrix of Convective Inertia (MCI) terms arising from the dynamic equations of motion. The simplicity of the matrices is emphasized here because for example, if the GIM is diagonal then the robot control is decoupled, i.e., each actuator can be controlled independently that will improve overall robot performances (speed and precision). Besides, the simulation in which, the inversion of the GIM is required becomes straight forward, as for a diagonal matrix its inversion is just another diagonal matrix whose diagonal elements are the reciprocals of the original matrix.

Earlier work on robot selection is based on the workspace and payload capacity, for example, those reported in Rivin (1988), Dorf and Nof (1988), and others. Another measure is the ease of changing the position and orientation of the end-effector, i.e., manipulability (Yoshikawa, 1998). All these concepts are kinematic ones, where the arm dynamics, i.e., mass and inertia of the links, are ignored. In this paper, one aspect of dynamics (Bhangale et al., 2001), namely, the simplicity of the Generalized Inertia Matrix (GIM) and the Matrix of Convective Inertia (MCI) terms associated with the dynamic equations of motion of a robot is used.

To derive the dynamic equations of motion of a robot manipulator, one of the two fundamental approaches, i.e., Newton-Euler (NE) (Fu et al., 1987) or Euler-Lagrange (EL) (Meirovitch, 1970) is used. In this paper, the dynamic model is developed using the NE equations and the concept of the Decoupled Natural Orthogonal Compliment (DeNOC) (Saha, 1999). The DeNOC concept allows one to write the elements of the GIM and MCI in explicit analytical forms. These forms led

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to the development of recursive dynamics algorithms for both the inverse and forward dynamics of serial (Saha, 1999; 2003), and parallel (Saha and Schiehlen, 2001) robotic systems. Besides, the explicit expressions can be used to simplify the GIM and MCI, which is explored in this paper for robot selection.

This paper is organized as follows: Section 2 outlines the dynamic modeling using the Decoupled Natural Orthogonal Compliment (DeNOC), where the key steps are pointed out for the purpose of robot selection. Section 3 shows the derivation of the GIM and MCI and computes the complexity in terms of floating point operations. Section 4 illustrates the robot selection methodology with the help of planar and spatial 3-link robots. Finally, conclusions are given in Section 5.

2 DYNAMIC MODELING USING THE DENOC

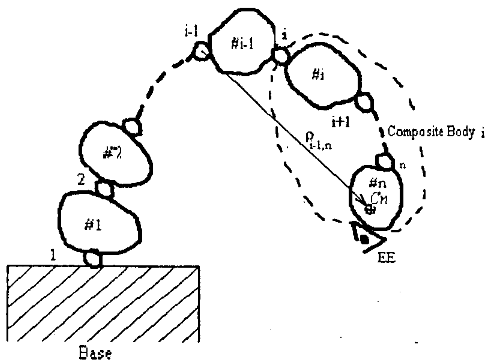
2.1 Kinematic Formulation

For an n-degree of freedom open-loop serial-robot, as shown in Fig.1, the n-dimensional joint rate vector, $\dot{\boldsymbol{\theta}}$, is defined as

$$\dot{\theta} = [\dot{\theta}_1, \ldots, \dot{\theta}_n]^T \qquad \ldots(1)$$

where θ_i, for i = 1,...,n, is the joint displacement of the i[th] joint. Accordingly, $\dot{\theta}_i$, is the joint rate. The twist and wrench vectors of the i[th] link, $\mathbf{t}_i$ and $\mathbf{w}_i$, are then introduced as

$$\mathbf{t}_i = \begin{bmatrix} \boldsymbol{\omega}_i \\ \mathbf{v}_i \end{bmatrix} \quad \text{and} \quad \mathbf{w}_i = \begin{bmatrix} \mathbf{n}_i \\ \mathbf{f}_i \end{bmatrix} \qquad \ldots(2)$$



1, 2, ...,n : Joints; #1, #2, ...#n : Links

Fig. 1: An n-link manipulator

where $\boldsymbol{\omega}_i$ and $\mathbf{v}_i$ are the 3-dimensional vectors of angular velocity and the linear velocity of the origin point O_i, of the i[th] body where it is coupled with its previous body in the chain, i.e., the (i-1)[st] body, respectively. Moreover, $\mathbf{n}_i$ and $\mathbf{f}_i$ are the 3-dimensional vectors denoting the resultant moment about O_i, and the resultant forces acting at O_i, respectively (Saha and Schiehlen, 2001). The 6n-dimensional vectors of generalized twist, $\mathbf{t}$, and the generalized wrench, $\mathbf{w}$, are defined next as

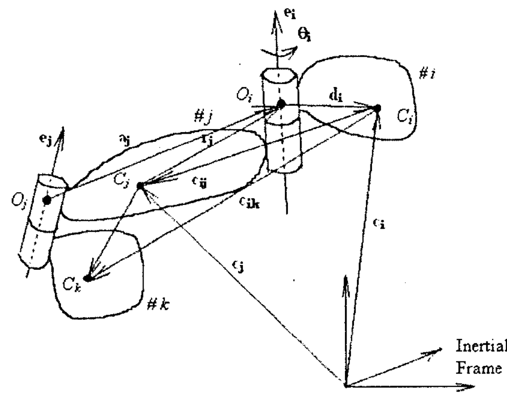


Fig. 2: A coupled system of three bodies

$$\mathbf{t} \equiv [\mathbf{t}_1^T, \mathbf{t}_2^T, \ldots, \mathbf{t}_n^T]^T; \ \mathbf{w} \equiv [\mathbf{w}_1^T, \mathbf{w}_2^T, \ldots, \mathbf{w}_n^T]^T \qquad \ldots(3)$$

From the kinematic constraints between the two successive links, say, #j and #i of Fig. 2, the twist, $\mathbf{t}_i$, can be expressed in terms of the twist of the previous body, $\mathbf{t}_j$, and the joint rate, $\dot{\theta}_i$, i.e.,

$$\mathbf{t}_i = \mathbf{A}_{ij}\mathbf{t}_j + \mathbf{p}_i\dot{\theta}_i \qquad \ldots(4)$$

where the 6 × 6 matrix, $\mathbf{A}_{ij}$ and the 6-dimensional vector, $\mathbf{p}_i$, are given by

$$\mathbf{A}_{ij} \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ -\mathbf{a}_{ij}\times\mathbf{1} & \mathbf{1} \end{bmatrix}; \mathbf{p}_i \equiv \begin{bmatrix} \mathbf{e}_i \\ \mathbf{0} \end{bmatrix} \text{ for revolute joint;}$$

$$\mathbf{p}_i \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{e}_i \end{bmatrix} \text{ for prismatic joint} \qquad \ldots(5)$$

$\mathbf{e}_i$ being the unit vector parallel to the axis of rotation (for revolute joint) or the direction of translation (for prismatic joint) of the i[th] joint. In eq. (5), ($\mathbf{a}_{ij} \times \mathbf{1}$) is the 3×3 cross-product tensor associated with the vector $\mathbf{a}_{ij}$, as indicated in Fig. 2, which when operates on the 3-dimensional Cartesian vector $\mathbf{x}$ results in a cross-product vector, i.e., $(\mathbf{a}_{ij}\times\mathbf{1})\mathbf{x} = \mathbf{a}_{ij}\times\mathbf{x}$. The 3×3 matrix, $\mathbf{1}$, is the identity matrix, whereas $\mathbf{O}$ and $\mathbf{0}$ are the 3×3 matrix and the 3-dimensional vector of zeros, respectively. Henceforth, the size of $\mathbf{1}$, $\mathbf{O}$, and $\mathbf{0}$ will be understood as those compatible with the matrix size where they appear. Equation (4) is now written for, i = 1, 2,...,n, and put in a compact form as

$$\mathbf{t} = \mathbf{T}\dot{\theta}, \quad \text{where } \mathbf{T} \equiv \mathbf{T}_l\mathbf{T}_d \qquad \ldots(6)$$

$\mathbf{T}$ being the 6nxn Natural Orthogonal Complement (NOC) matrix (Angeles and Lee, 1988), whereas the 6nx6n and 6nxn matrices, $\mathbf{T}_l$ and $\mathbf{T}_d$, respectively, are the Decoupled NOC (DeNOC) matrices (Saha, 1999). The structures of the DeNOC matrices, $\mathbf{T}_l$ and $\mathbf{T}_d$, are shown below:

$$\mathbf{T}_l \equiv \begin{bmatrix} \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{A}_{21} & \mathbf{1} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{n,1} & \mathbf{A}_{n-1,2} & \cdots & \mathbf{1} \end{bmatrix}$$

and $$\mathbf{T}_d \equiv \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{p}_n \end{bmatrix} \quad \text{...(7)}$$

where $\mathbf{A}_{ij}$ and $\mathbf{p}_i$, for i, j = 1,...,n, are defined in eq. (5). The concept of the DeNOC is useful for the development of efficient recursive dynamics algorithms, both inverse and forward dynamics, as shown in Saha (1999, 2003), and Saha and Schiehlen (2001).

2.2 Dynamic Modeling

For the system under study, as shown in Fig. 1, if m_i is the mass of the i^{th} link and $\mathbf{I}_i$ denotes the 3×3 inertia tensor of the i^{th} link about its origin point, O_i, then the uncoupled Newton-Euler equations (NE) governing the motion of the i^{th} link can be written as

$$\mathbf{M}_i \dot{\mathbf{t}}_i + \mathbf{W}_i\mathbf{M}_i\mathbf{E}_i\mathbf{t}_i = \mathbf{w}_i \quad \text{...(8)}$$

where the 6 × 6 matrix, $\mathbf{M}_i$ and $\mathbf{W}_i$ for the i^{th} link are given as (Saha and Schiehlen, 2001)

$$\mathbf{M}_i \equiv \begin{bmatrix} \mathbf{I}_i & m_i\mathbf{d}_i \times \mathbf{1} \\ -m_i\mathbf{d}_i \times \mathbf{1} & m_i\mathbf{1} \end{bmatrix};$$

$$\mathbf{W}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \times \mathbf{1} & \mathbf{O} \\ \mathbf{O} & \boldsymbol{\omega}_i \times \mathbf{1} \end{bmatrix}; \; \mathbf{E}_i = \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \quad \text{...(9)}$$

in which $\mathbf{I}_i \equiv \mathbf{I}_i^c - \mathbf{d}_i \times (m_i\mathbf{d}_i \times \mathbf{1})$ -- $\mathbf{I}_i^c$ being the 3×3 inertia tensor about the mass centre of the i^{th} body, C_i, $\mathbf{d}_i\times\mathbf{1}$ and $\boldsymbol{\omega}_i\times\mathbf{1}$ are the 3×3 cross-product tensors associated with the vector $\mathbf{d}_i$ shown in Fig. 2, and the angular velocity, ω_i, respectively. The wrench vector, $\mathbf{w}_i$, is already defined in eq. (2). Equation (9) is written for all the n links, i.e., i = 1, 2,..., n, and expressed in compact form as

$$\mathbf{M}\,\dot{\mathbf{t}} + \mathbf{WMEt} = \mathbf{w} \quad \text{...(10)}$$

where **M** is the 6n×6n generalized mass matrix, **W** is the 6n×6n generalized matrix of the angular velocities, **w** is the 6n-dimensional generalized vector of wrench, and **t** is the 6n-dimensional generalized vector of twist. Vectors **t** and **w** are defined in eq. (3), whereas the matrices, **M** and **W**, are defined as follows:

$\mathbf{M} \equiv \text{diag.}[\mathbf{M}_1, \dots, \mathbf{M}_n]$; $\mathbf{W} \equiv \text{diag.}[\mathbf{W}_1, \dots, \mathbf{W}_n]$; $\mathbf{E} \equiv \text{diag.}[\mathbf{E}_1, \dots, \mathbf{E}_n]$.

Premultiplying eq. (10) with the transpose of the NOC matrix, i.e., $\mathbf{T}^T$, one gets n independent dynamic equations of motion of the coupled system, namely,

$$\mathbf{T}^T(\mathbf{M}\,\dot{\mathbf{t}} + \mathbf{WMEt}) = \mathbf{T}^T(\mathbf{w}^E + \mathbf{w}^C) \quad \text{...(11)}$$

where **w** is substituted by $\mathbf{w} \equiv \mathbf{w}^E + \mathbf{w}^C$, $\mathbf{w}^E$ and $\mathbf{w}^C$ being the 6n-dimensional vectors of external and constraint wrenches, respectively. The term $\mathbf{T}^T\mathbf{w}^C$ in eq. (11) vanishes, as the constraint wrench produces no work. Substitution of the expression of $\mathbf{T} = \mathbf{T}_l\mathbf{T}_d$ from eq. (6) and its time derivative, $\dot{\mathbf{T}} = \mathbf{T}_l\dot{\mathbf{T}}_d + \dot{\mathbf{T}}_l\mathbf{T}_d$, into eq. (11), results in the following form of the dynamic equations of motion:

$$\mathbf{I}\,\ddot{\boldsymbol{\theta}} + \mathbf{C}\,\dot{\boldsymbol{\theta}} = \boldsymbol{\tau} \quad \text{...(12)}$$

which is nothing but the Euler-Lagrange equations of motion (Angeles and Lee, 1988). In eq. (12),

$\mathbf{I} \equiv \mathbf{T}_d^T\tilde{\mathbf{M}}\mathbf{T}_d$: the n × n generalized inertia matrix;

$\mathbf{C} \equiv \mathbf{T}_d^T(\mathbf{T}_l^T\mathbf{M}\dot{\mathbf{T}}_l + \tilde{\mathbf{M}}\mathbf{W} + \tilde{\dot{\mathbf{M}}})\mathbf{T}_d$: the n×n matrix of convective inertia (MCI) terms;

$\boldsymbol{\tau} \equiv \mathbf{T}_d^T\tilde{\mathbf{w}}^E$: the n-dimensional vector of generalized forces due to driving forces / torques and those resulting from the gravity and dissipation.

The 6n × 6n matrices, $\tilde{\mathbf{M}}, \tilde{\dot{\mathbf{M}}}$ and the 6n-dimensional vector $\tilde{\mathbf{w}}^E$ are given by

$$\tilde{\mathbf{M}} \equiv \mathbf{T}_l^T\mathbf{M}\mathbf{T}_l; \; \tilde{\dot{\mathbf{M}}} \equiv \mathbf{T}_l^T\mathbf{WMET}_l; \text{and } \tilde{\mathbf{w}}^E \equiv \mathbf{T}_l^T\mathbf{w}^E$$

Using the expressions for $\mathbf{T}_l$ and $\mathbf{T}_d$ from eq.(7), the elements of the n × n matrices **I**, **C** and, the n-dimensional vector, **τ**, as appear after eq.(12), i.e., i_{ij}, c_{ij} and τ_i, respectively are written analytically as (Saha, 1999),

$$i_{ij} \equiv \mathbf{p}_i^T\tilde{\mathbf{M}}_i\mathbf{A}_{ij}\mathbf{p}_j$$

$$c_{ij} \equiv \mathbf{p}_i^T(\mathbf{A}_{ji}^T\tilde{\mathbf{M}}_j\mathbf{W}_j + \mathbf{A}_{j+1,i}^T\tilde{\mathbf{H}}_{j+1,j} + \mathbf{A}_{ji}^T\tilde{\dot{\mathbf{M}}}_j)\mathbf{p}_j \quad \text{...if } i \le j$$

$$c_{ij} \equiv \mathbf{p}_i^T(\tilde{\mathbf{M}}_i\mathbf{A}_{ij}\mathbf{W}_j + \tilde{\mathbf{H}}_{ij} + \tilde{\dot{\mathbf{M}}}_i\mathbf{A}_{ij})\mathbf{p}_j \quad \text{...otherwise}$$

$$\tau_i \equiv \mathbf{p}_i^T\tilde{\mathbf{w}}_i \quad \text{...(13)}$$

where the matrix, $\tilde{\mathbf{H}}_{ij}$, in the expression of c_{ij}, is given as $\tilde{\mathbf{H}}_{ij} \equiv \tilde{\mathbf{M}}_i\dot{\mathbf{A}}_{ij} + \mathbf{A}_{i+1,i}^T\tilde{\mathbf{H}}_{i+1,i}\mathbf{A}_{ij}$, in which $\dot{\mathbf{A}}_{ij}$ is the time derivative of the 6×6 matrix, $\mathbf{A}_{ij}$, defined in eq. (5). Moreover, the 6-dimensional vector $\tilde{\mathbf{w}}_i$ is given by

$$\tilde{\mathbf{w}}_i \equiv \mathbf{w}_i + \mathbf{A}_{i,i+1}^T\tilde{\mathbf{w}}_{i+1}, \text{ where } \tilde{\mathbf{w}}_n = \mathbf{w}_n \quad \text{...(14)}$$

In eq. (13), the 6 × 6 matrix, $\tilde{\mathbf{M}}_i$, has the following features which plays an important role in deciding many of the dynamic characteristics of the robot including the simplification of the GIM and MCI:

1) Matrix $\tilde{\mathbf{M}}_i$ can be recursively computed as

$$\tilde{\mathbf{M}}_i \equiv \mathbf{M}_i + \mathbf{A}_{i+1,i}^T\tilde{\mathbf{M}}_{i+1}\mathbf{A}_{i+1,i} \quad \text{for } i = n, n-1, \dots, 1$$

where $\tilde{\mathbf{M}}_n \equiv \mathbf{M}_n$ since there is no $(n+1)^{st}$ link, i.e., $\tilde{\mathbf{M}}_{n+1} \equiv \mathbf{O}$. Moreover, $\tilde{\mathbf{M}}_{n-1} \equiv \mathbf{M}_{n-1} + \mathbf{A}_{n,n-1}^T\mathbf{M}_n\mathbf{A}_{n,n-1}$.

2) Matrix $\tilde{\mathbf{M}}_i$ has the physical interpretation, i.e., it represents the mass matrix of the "composite body", i, formed by rigidly joining the bodies, i,...,n, as indicated in Fig. 1.

3) The elements of $\tilde{\mathbf{M}}_i$ and the GIM or MCI can be made vanish or constant with suitable choice of the link masses and geometries. For example, if any two joints are prismatic which are orthogonal and intersecting, the corresponding inertia element, i_{ij} can be proven to be zero.

For the planar case, the above complexity can be further simplified by eliminating where zero components of a vector or a matrix. The computational costs are for an n-link n-revolute joint planar robot

GIM: $(3.5n^2 + 11.5n - 7)M\,(2n^2 + 9n - 7)A$....(15a)

MCI: $(7n^2 + 13n + 4)M\,(4n^2 + 13n + 2)A$. ...(15b)

Furthermore, if there are prismatic joint the computational costs for both spatial and planar cases further reduces which is very obvious from the expressions given in eq. (13). Now the selection

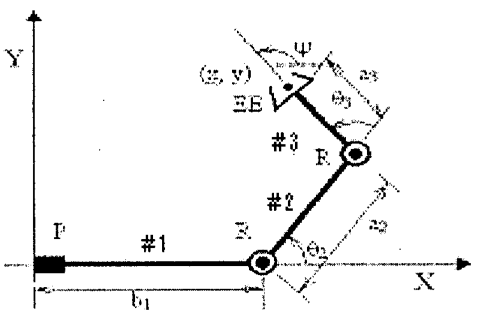


Fig. 3: A 3-link 2R-1P (PRR) planar manipulator

3 PROPOSED ROBOT SELECTION METHODOLOGY

In this section, proposed robot selection methodology based on the DeNOC based dynamics presented in Section 2 is illustrated with the help of 3-link 3-degree of freedom (DOF) planar and spatial robot arms. Based on the dynamic model given by eqs. (12) and (13), computational complexities for the GIM and MCI of an n-link, n-revolute joint serial robot can be computed as (Bhangale, 2003)

GIM: $(11n^2 + 34n - 18)M\,(7n^2 + 37n - 18)A$....(16a)

MCI: $(14n^2 + 22n + 4)M\,(13.5n^2 + 55.5n - 65.5)A$. ...(16b)

where 'M' and 'A' stand for multiplications / divisions and additions / subtractions, respectively. Note that the GIM complexity is less than the value reported by Walker and Orin (1982), i.e., $(12n^2 + 56n - 27)M\ (7n^2 + 67n - 53)A$, whereas for MCI complexity value is not available for comparison.

3.1 Selection of a Planar 3-DOF Robot

A robot performing a planar task requires 3-DOF, two for positions and one for orientation. One can have choices of a robot with the following joint combinations: (a) three revolute, 3R; (b) two revolute one prismatic, 2R-1P; and (c) two prismatic one revolute, 2P-1R. In case of choice (a), there is no change in the kinematic configuration when the joints are interchanged, i.e., no change in computational counts, i.e., for n = 3 in eq. (15), $59M$ $38A$ for GIM and $106M$ $77A$ for the MCI, are required. In choices (b) and (c), however, one can have the following choices: RRP, RPR, PRR (Fig. 3) for (b), and PPR, PRP, RPP for (c). Using the proposed methodology one can select a suitable 2R-1P or 2P-1R architecture based on the simplicity of the GIM and MCI. Here, 2R-1P, i.e., two revolute and one prismatic joint combination is taken for illustration purposes. The computational complexity of the GIM, and MCI terms for RRP, RPR, PRR are evaluated, from eq. (15) which are tabulated in Table 1. Note that the simplifications due to the orthogonality of the prismatic joint with the other revolute joints are taken into account. Differences in the results are due to the placements of the prismatic joint at the location 1, 2 or 3.

Table 1: Computation Complexities of 2R-1P robots

Type	GIM Computation	MCI Computation	Total
RRP	$46M$; $29A$	$76M$; $42A$	$122M$; $71A$
RPR	<u>$34M$; $22A$</u>	$88M$; $39A$	$122M$; $61A$
PRR	$42M$; $27A$	<u>$53M$; $29A$</u>	<u>$95M$; $56A$</u>

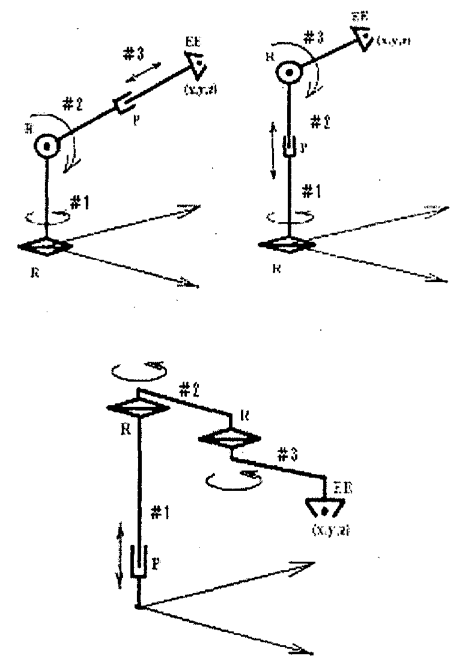
M: Multiplication / Division; A: Addition / Subtraction.
___ (Underline): Minimum Values.

Based on the results in Table 1, PRR, as shown in Fig. 3(c), has minimum complexity, which would provide higher speed in control and simulation.

3.2 Spatial 3-DOF Robot Arm

Three-DOF spatial robot arms with two revolute and one prismatic joint are shown in Fig. 4 (a)-(c). Two of which, namely, those in Fig. 4(a) and (c) are the Stanford and RTX robot arms, respectively. While the Stanford arm has spherical workspace, RTX is of SCARA type. In order to consider a case where the prismatic joint is at location 2, a third type is conceived here. Computational complexities for the Generalized Inertia Matrix (GIM) and Matrix of Convective Inertia (MCI) terms for all these robot arms are tabulated in Table 2, which are based on eq. (16). Similar to the planar case, simplicity due to the orthogonal positions of the joint axes is also taken into account. Based on the total minimum computational complexity PRR configuration (RTX Robot), shown in Fig. 4(c) is selected.

dynamics algorithms required for robot motion control.



(a

Fig. 4: The 3-link 2P-1R spatial manipulators

Table 2: Computational Complexities of spatial 2R-1P robots

Type	GIM Computation	MCI Computation	Total
RRP (Stanford)	170M; 154A	522M; 352A	692M; 506A
RPR	153M; 148A	556M; 415A	709M; 563A
PRR (RTX)	142M; 138A	389M; 317A	531M; 455A

M: Multiplication/Division; A: Addition/Subtraction.
___ (Underline): Minimum Values.

4 CONCLUSIONS

Robot selection based on the computational complexities of the Generalized Inertia Matrix (GIM), and Matrix of Convective Inertia (MCI) terms, **I** and **C** of eq. (13), respectively, arising from the dynamic equations of motion of the manipulator at hand, is used to select robot architecture. The concept is illustrated with the help of 3-link 3-DOF planar and spatial robots with two revolute and one prismatic joint. In the planar case PRR configuration, and in the spatial case PRR configuration (RTX Robot), as shown in Figs. 3 and 4(c), respectively, provided the most economic computational counts. Hence, it is anticipated that these robots will provide fast control and simulation algorithms. This has been verified through the CPU time requirement of the inverse dynamics algorithms required for robot motion control.

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