DeNOC-based Dynamic Modelling of Multibody Systems

(5th in a series)

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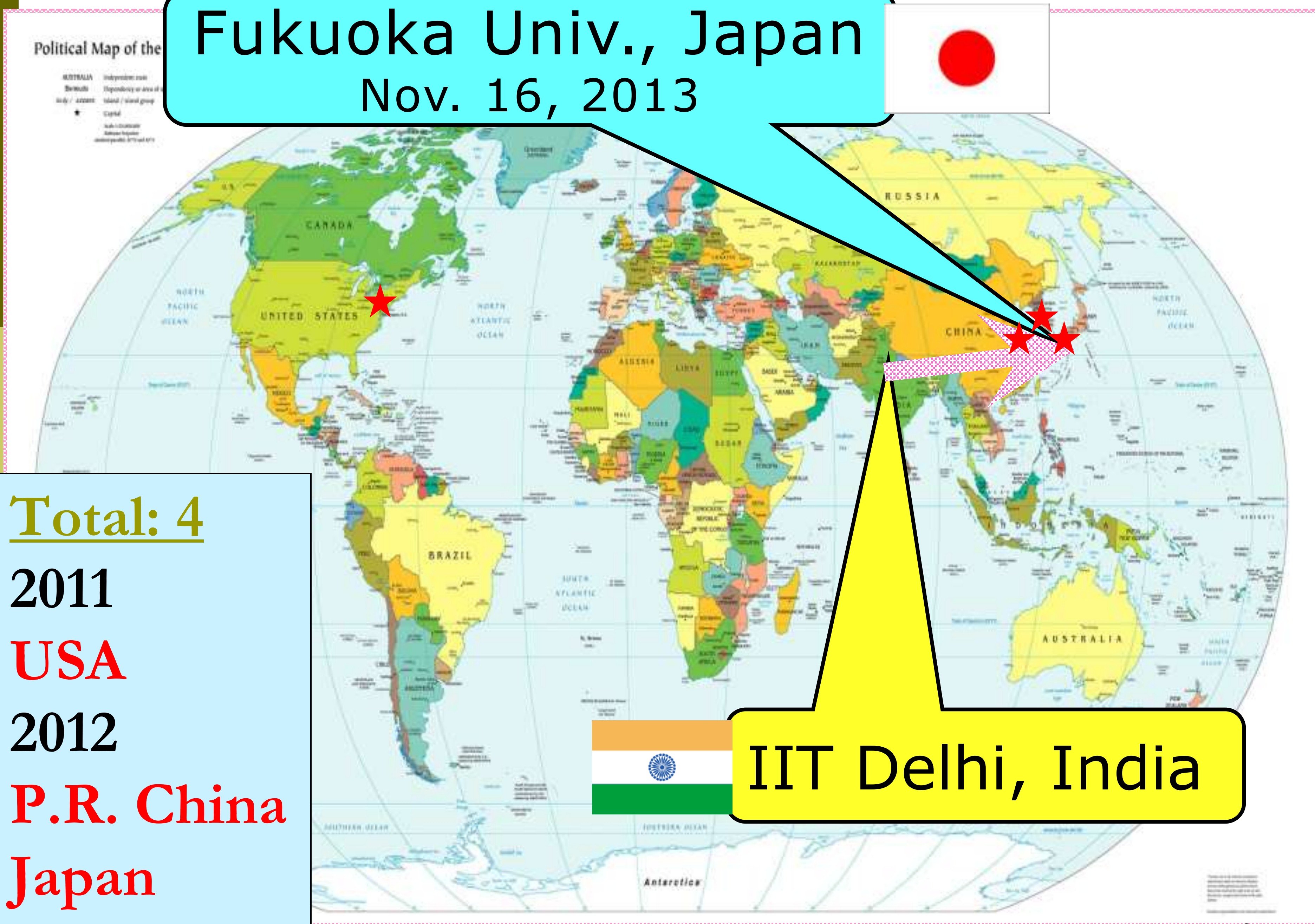
Fukuoka Univ., Japan

Nov. 16, 2013



IIT Delhi, India

<u>Total: 4</u>

2011

USA

2012

P.R. China

Japan

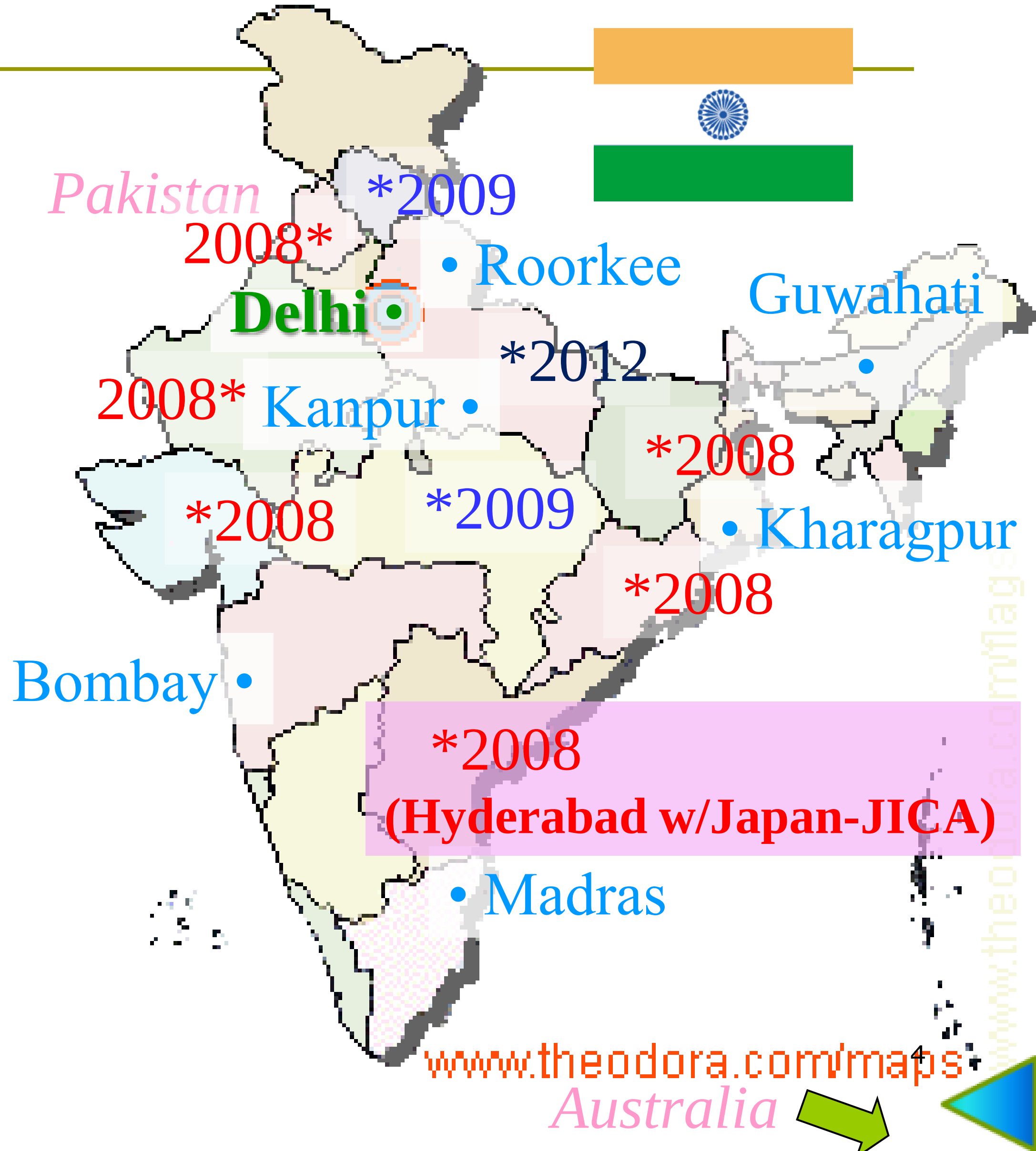
S. Korea

Plan of Presentation

- Introduction
- Serial systems
  - RoboAnalyzer
- Tree-type systems
  - Modeling & Simulation
- Closed-loop systems
- Multibody Dynamics for Rural Applications (MuDRA)
- Conclusions

Introduction to IIT Delhi

- Indian Institute of Technology (IIT)
  - 7 + 9 new IITs = 16
  - Centers of Excellence
- **IIT Delhi** (1961): Celebrating 50 Years
  - 320 acres
  - 13 Departments
  - 11 Centers
  - 3 Schools
  - 8000 UG/PG/PH. D



PAR Lab.@IIT Delhi

(10 Faculty + 20 Students)

Programme for Autonomous Robotics
(Sponsored by BARC/BRNS)

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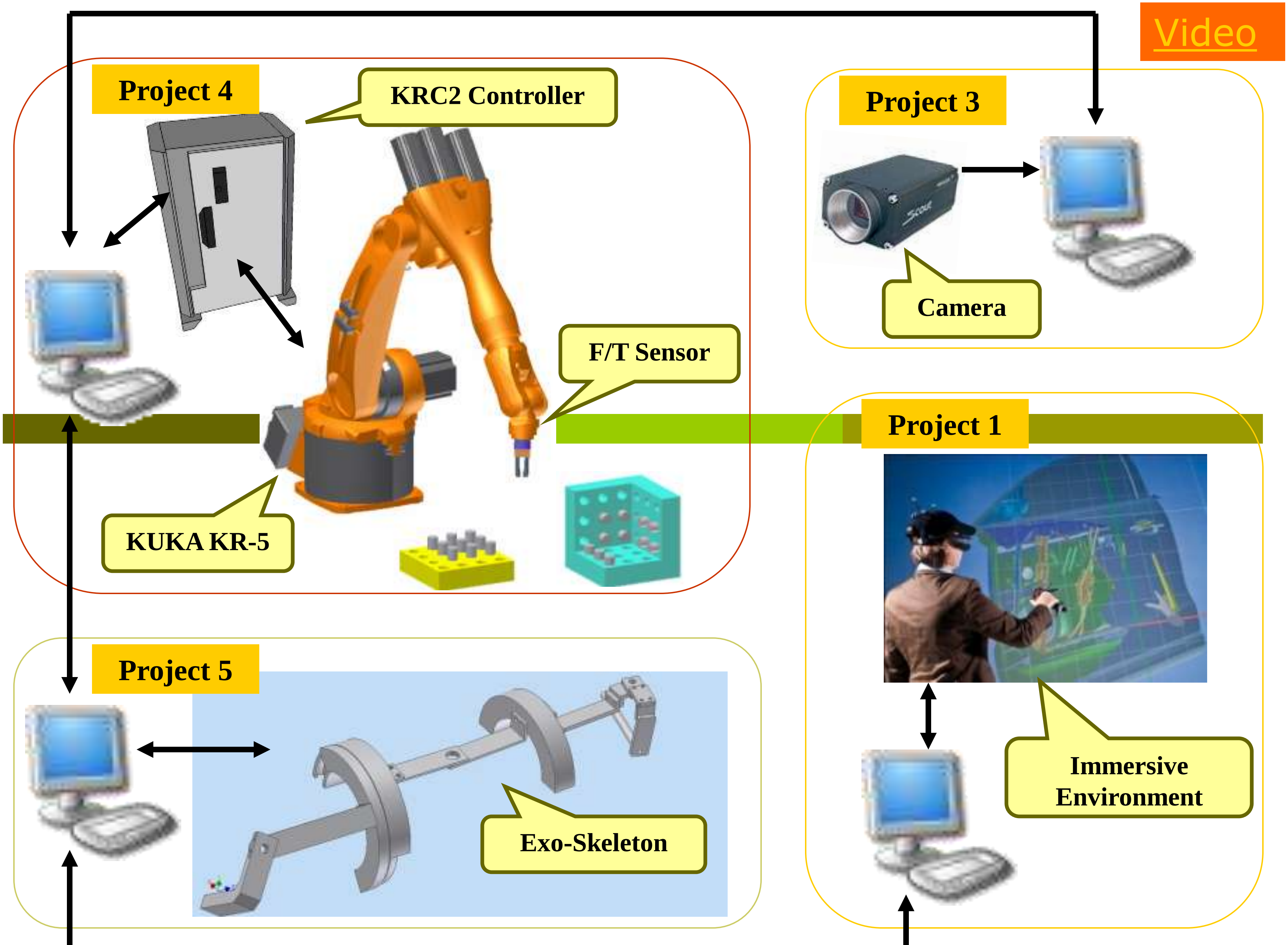

Layout of Four Interlinked Projects

Mechatronics Laboratory, IIT Delhi

Estd. 2001

Robocon

Annual Report 2011

Brochure

Evolution of DeNOC

2013 Long Chain (With IIT Madras)

2012 Book by Springer & ReDySim + Tree-type (Ph. D)

2011 Engine Cam (M.S.-IITM)

2009 RoboAnalyzer Software (IITD)

2007 Book by Springer + Closed-loop (Ph. D); Flexible serial (Ph.D)

2006 Machine Tool (Ph. D)

2005 3-DOF parallel (McGill, Canada)

2003 RIDIM (IITD)

2000 Parallel robots (Univ. of Stuttgart, Germany)

1999 Industrial Robots (IITD)

1995 Space robots (Toshiba, Japan)

Serial Chain: DeNOC-Based

A Decomposition of the Manipulator Inertia Matrix

Subir Kumar Saha

Abstract—A decomposition of the manipulator inertia matrix is essential, for example, in forward dynamics, where the joint accelerations are solved from the dynamical equations of motion. To do this, unlike a numerical algorithm, an analytical approach is suggested in this paper. The approach is based on the symbolic Gaussian elimination of the inertia matrix that reveal recursive relations among the elements of the resulting matrices. As a result, the decomposition can be done with the complexity of order n, $\mathcal{O}(n)$—n being the degrees of freedom of the manipulator—, as opposed to an $\mathcal{O}(n^3)$ scheme, required in the numerical approach. In turn, $\mathcal{O}(n)$ inverse and forward dynamics algorithms can be developed. As an illustration, an $\mathcal{O}(n)$ forward dynamics algorithm is presented.

Index Terms— Articulated-body inertia, Kalman filtering, reverse Gaussian elimination (RGE), serial manipulator, symbolic decomposition.

I. INTRODUCTION

The inertia matrix of a robotic manipulator or the generalized inertia matrix (GIM) arises from the robot's dynamic equations of motion. The decomposition of the GIM is required, for example,

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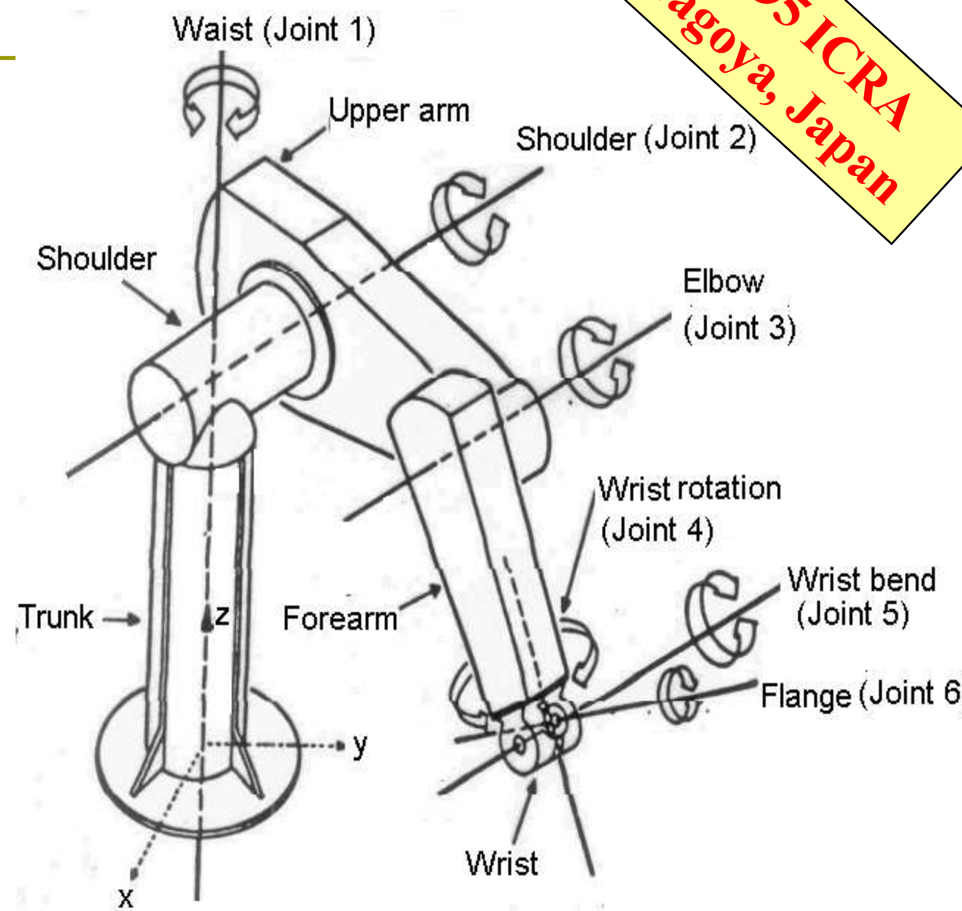
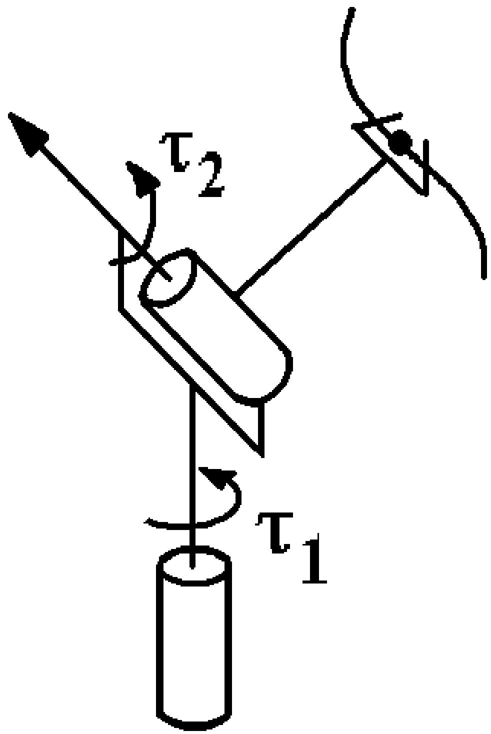
1997 IEEE Trans. on Rob. & Aut.
V. 13, N. 2, Apr., pp. 301-304

1995 ICRA
Nagoya, Japan



PUMA Robot

Inverse vs. Forward Dynamics

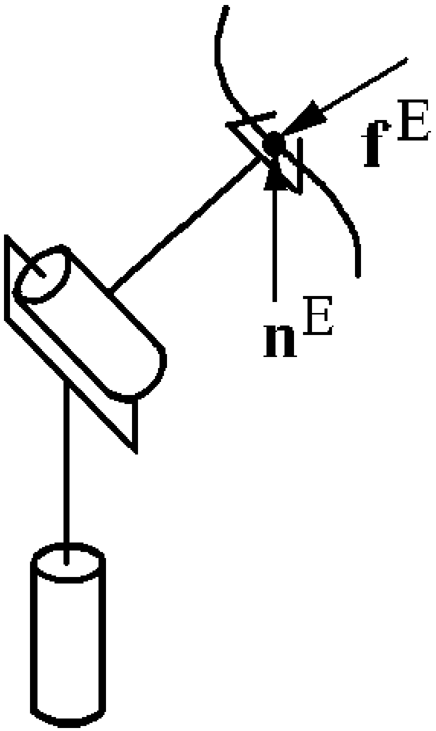


Inverse Dynamics

Find joint torques/forces for given joint motions and end-effector moment/force

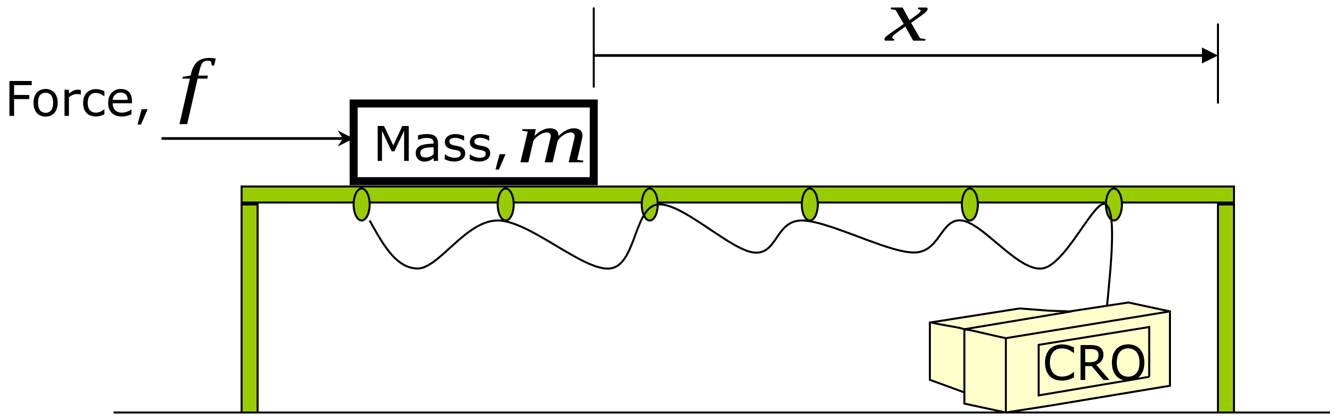
Forward Dynamics

Find end-effector motion for known joint torques/forces



Simple System

Actually



Newton's 2nd law: $f = m\ddot{x}$ → Modelling

Given $m, \ddot{x}$; Find f → Inverse dynamics

Given m, f ; Find $\ddot{x} = \dfrac{f}{m}$ → Forward dynamics

$\dot{x} = \int \ddot{x}dt; \quad x = \int \dot{x}dt$ → Integrations

Methods

- Newton-Euler (NE)

  Euler's: $\mathbf{I}_i\dot{\omega}_i + \omega_i \times \mathbf{I}_i\omega_i = \mathbf{n}_i$

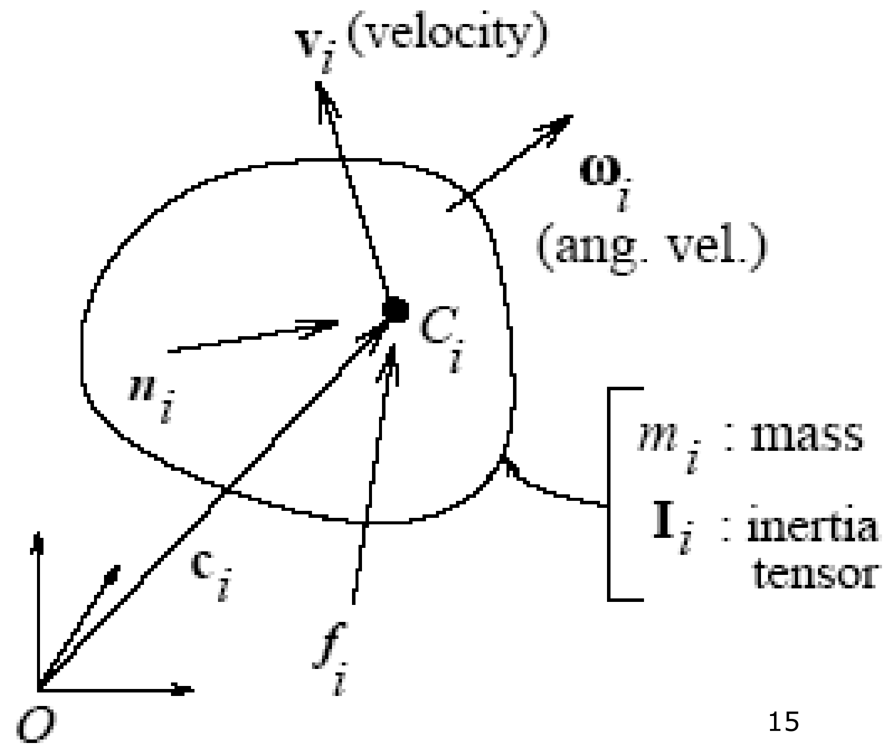
  Newton's: $m_i\dot{\mathbf{v}}_i = \mathbf{f}_i$

  $\Rightarrow \mathbf{M}_i\dot{\mathbf{t}}_i + \dot{\mathbf{M}}_i\mathbf{t}_i = \mathbf{w}_i$

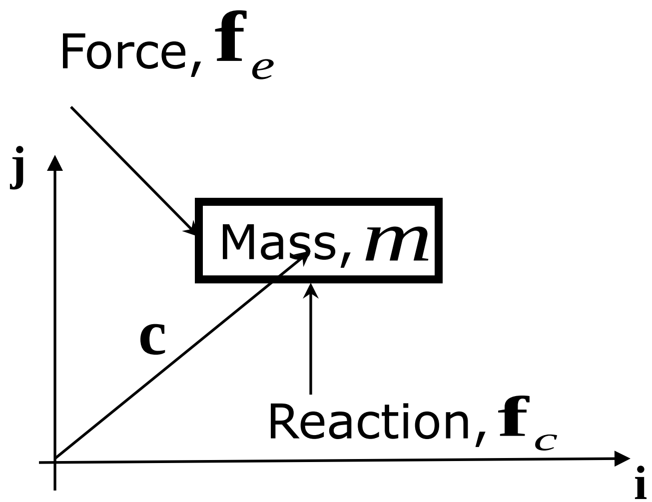
- Euler-Lagrange (EL)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{\theta}}}\right) - \frac{\partial L}{\partial \boldsymbol{\theta}} = \boldsymbol{\tau}$$

- Kane's, Hamilton's …
- Orthogonal Complement based, e.g., *Decoupled Natural Orthogonal Complement (DeNOC)*



DeNOC for Simple System



Free-body diagram

Note that

$$[\mathbf{i}]^T \mathbf{f}_c =$$

$$[\mathbf{i}]^T \ f_c[\mathbf{j}] = 0$$

Newton's 2nd law: $\mathbf{f}_e + \mathbf{f}_c = m\ddot{\mathbf{c}}$

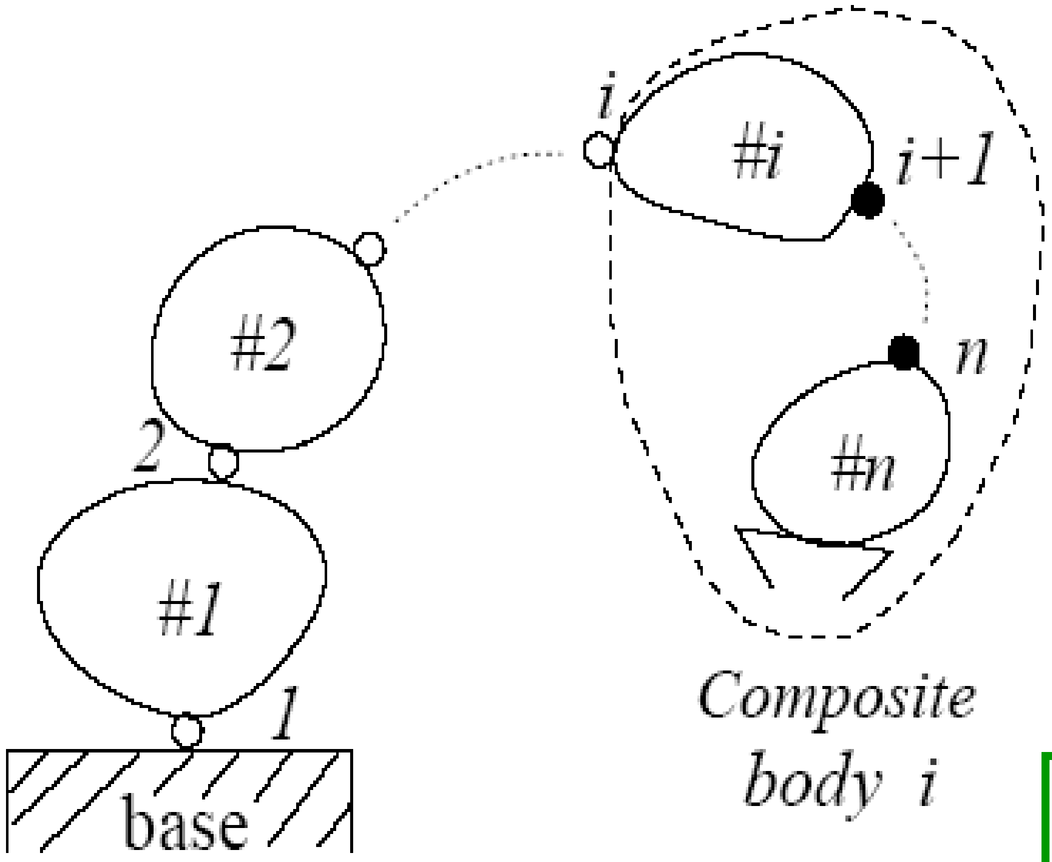
Velocity constraint: $\dot{\mathbf{c}} = [\mathbf{i}]\dot{x}$

Natural Orthogonal Complement: $[\mathbf{i}]$

Constrained motion (Euler-Lagrange):

$$[\mathbf{i}]^T(\mathbf{f}_e + \mathbf{f}_c) = [\mathbf{i}]^T m\ddot{\mathbf{c}} \Rightarrow f = m\ddot{x}$$

Uncoupled NE Equations



$$\mathbf{M} \equiv \mathrm{diag}[\mathbf{M}_1, \cdots, \mathbf{M}_n]$$

$$\dot{\mathbf{t}} \equiv [\dot{\mathbf{t}}_1^T, \cdots, \dot{\mathbf{t}}_n^T]^T$$

$$\dot{\mathbf{M}} \equiv \mathrm{diag}[\dot{\mathbf{M}}_1, \cdots, \dot{\mathbf{M}}_n]$$

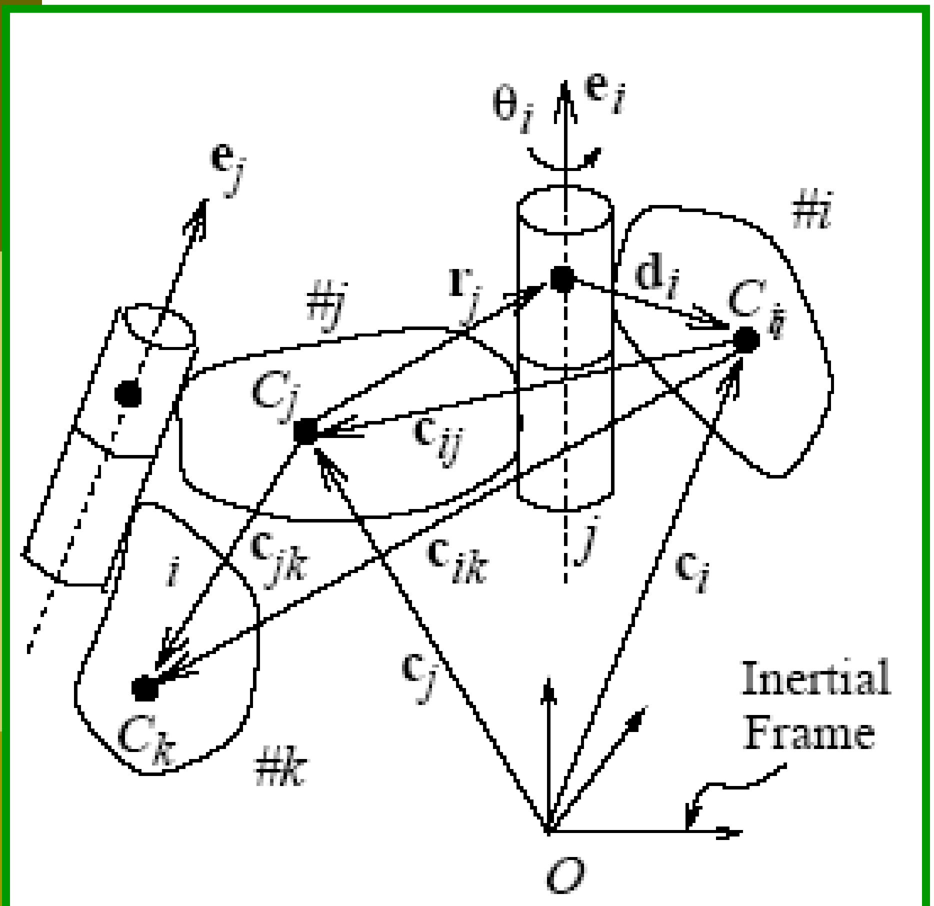
$$\mathbf{t} \equiv [\mathbf{t}_1^T, \cdots, \mathbf{t}_n^T]^T$$

$$\mathbf{w} \equiv [\mathbf{w}_1^T, \cdots, \mathbf{w}_n^T]^T$$

$$\boxed{\mathbf{M}\dot{\mathbf{t}} + \dot{\mathbf{M}}\mathbf{t} = \mathbf{w}}$$

- The 6*n* uncoupled equations of motion

Velocity Constraints: DeNOC Matrices



$$\boldsymbol{\omega}_i = \boldsymbol{\omega}_j + \dot{\theta}_i \mathbf{e}_i$$

$$\mathbf{v}_i = \mathbf{v}_j + \boldsymbol{\omega}_j \times \mathbf{r}_j + \boldsymbol{\omega}_i \times \mathbf{d}_i$$

$$\mathbf{t}_i = \mathbf{B}_{ij}\mathbf{t}_j + \mathbf{p}_i \dot{\theta}_i$$

$$\mathbf{B}_{ij}\mathbf{B}_{jk} = \mathbf{B}_{ik}$$

$$\mathbf{B}_{ii} = \mathbf{1}, \qquad \text{and} \qquad \mathbf{B}_{ij}^{-1} = \mathbf{B}_{ji}$$

$\mathbf{B}_{ij}$: the $6n \times 6n$ <u>twist-propagation</u> matrix

$\mathbf{p}_i$: the $6n$-dimensional joint-rate propagation vector or <u>twist generator</u>

Definition: DeNOC Matrices

$$\mathbf{t} \equiv [\mathbf{t}_1^T, \cdots, \mathbf{t}_n^T]^T \quad \dot{\boldsymbol{\theta}} \equiv [\dot{\theta}_1, \cdots, \dot{\theta}_n]^T$$

$$\boxed{\mathbf{t} = \mathbf{N}\dot{\boldsymbol{\theta}}, \qquad \text{where} \quad \mathbf{N} \equiv \mathbf{N}_l \mathbf{N}_d}$$

$$\mathbf{N}_l \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{B}_{21} & \mathbf{1} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{n1} & \mathbf{B}_{n2} & \cdots & \mathbf{1} \end{bmatrix} \quad \text{and} \quad \mathbf{N}_d \equiv \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_n \end{bmatrix}$$

- **$\mathbf{N} \equiv \mathbf{N}_l \mathbf{N}_d$:** the $6n \times n$ Decoupled Natural Orthogonal Complement

Coupled Equations

$$\mathbf{N}^T(\mathbf{M}\dot{\mathbf{t}} + \dot{\mathbf{M}}\mathbf{t}) = \mathbf{N}^T(\mathbf{w}^W + \mathbf{w}^C) \Leftarrow \mathbf{t}^T\mathbf{w}^C = \dot{\boldsymbol{\theta}}^T\mathbf{N}^T\mathbf{w}^C = \mathbf{0},$$

$$\Downarrow$$

$$\mathbf{N}^T(\mathbf{M}\dot{\mathbf{t}} + \dot{\mathbf{M}}\mathbf{t}) = \mathbf{N}^T\mathbf{w}^W$$

$$\Downarrow$$

$$\boxed{\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} = \boldsymbol{\tau}}$$

$$\mathbf{I} \equiv \mathbf{N}^T\mathbf{M}\mathbf{N} \equiv \mathbf{N}_d^T\tilde{\mathbf{M}}\mathbf{N}_d$$

$$\mathbf{C} \equiv \mathbf{N}^T(\mathbf{M}\dot{\mathbf{N}} + \dot{\tilde{\mathbf{M}}}\mathbf{N}) \equiv \mathbf{N}_d^T\tilde{\mathbf{M}}'\mathbf{N}_d$$

$$\boldsymbol{\tau} \equiv \mathbf{N}^T\mathbf{w}^W \equiv \mathbf{N}_d^T\tilde{\mathbf{w}}^W;$$

- *n* coupled Euler-Lagrange equations

  - *no partial differentiation*

Recursive Expressions

- For the $n \times n$ GIM, each element

$$I_{ij} = I_{ji} = \mathbf{p}_i^T \tilde{\mathbf{M}}_i \mathbf{B}_{ij} \mathbf{p}_j$$

$$\boxed{\tilde{\mathbf{M}}_i = \mathbf{M}_i + \mathbf{B}_{i+1,i}^T \tilde{\mathbf{M}}_{i+1} \mathbf{B}_{i+1,i} \quad \text{where} \quad \tilde{\mathbf{M}}_n \equiv \mathbf{M}_n}$$

Composite body mass matrix

- For the $n \times n$ MCI, each element

$$C_{ij} = \begin{cases} \mathbf{p}_i^T (\mathbf{B}_{ji}^T \tilde{\mathbf{M}}_j \mathbf{W}_j + \mathbf{B}_{j+1,i}^T \tilde{\mathbf{H}}_{j+1,j} + \dot{\tilde{\mathbf{M}}}_j) \mathbf{p}_j & \text{if } i \le \tilde{\mathbf{M}}_{n+1} = \tilde{\mathbf{H}}_{n+1,n} = \mathbf{O} \\ \mathbf{p}_i^T (\tilde{\mathbf{M}}_i \mathbf{B}_{ij} \mathbf{W}_j + \tilde{\mathbf{H}}_{ij} + \dot{\tilde{\mathbf{M}}}_i) \mathbf{p}_j & \text{otherwise} \end{cases}$$

- For the $n \times n$ generalized forces

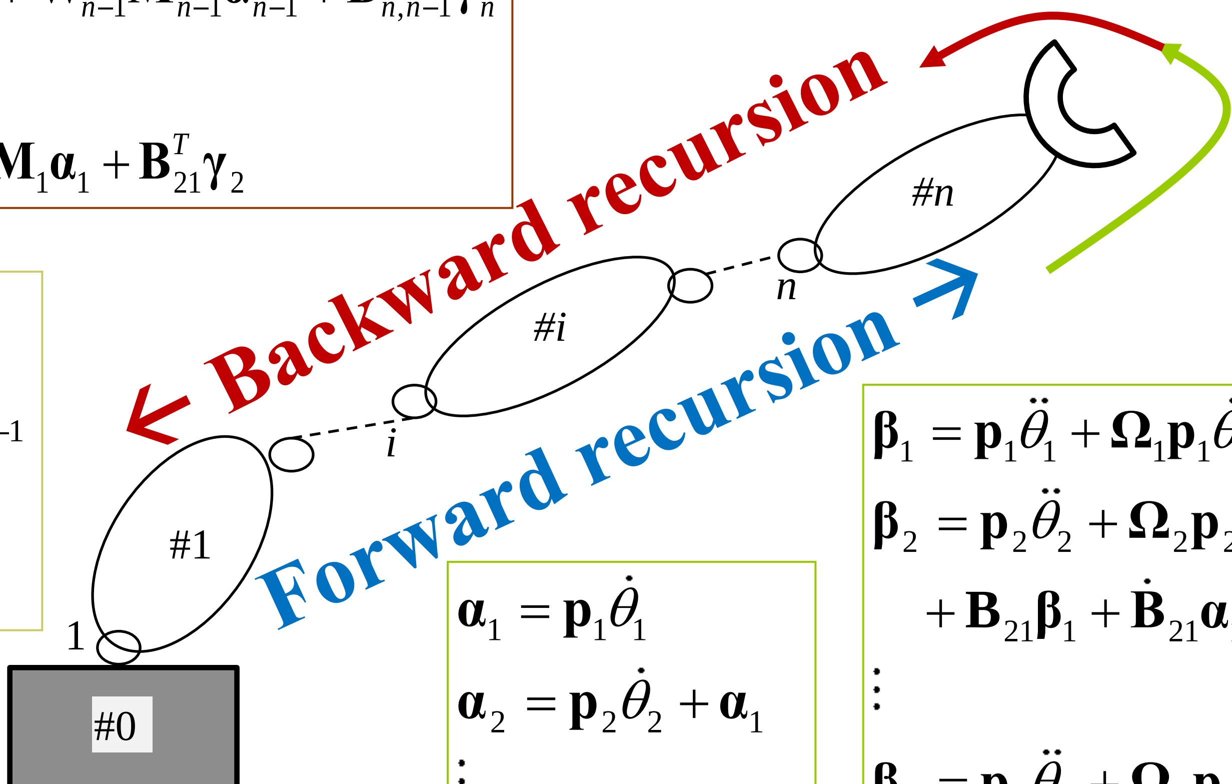
$$\tau_i = \mathbf{p}_i^T \tilde{\mathbf{w}}_i^W \qquad \boxed{\tilde{\mathbf{w}}_i^W = \mathbf{w}_i^W + \mathbf{B}_{i+1,i}^T \tilde{\mathbf{w}}_{i+1}^W}$$

Recursive Inverse Dynamics Algorithm

Saha (1999): ASME

$$\boldsymbol{\gamma}_n = \mathbf{M}_n\boldsymbol{\beta}_n + \mathbf{W}_n\mathbf{M}_n\boldsymbol{\alpha}_n$$
$$\boldsymbol{\gamma}_{n-1} = \mathbf{M}_{n-1}\boldsymbol{\beta}_{n-1} + \mathbf{W}_{n-1}\mathbf{M}_{n-1}\boldsymbol{\alpha}_{n-1} + \mathbf{B}^T_{n,n-1}\boldsymbol{\gamma}_n$$
$$\vdots$$
$$\boldsymbol{\gamma}_1 = \mathbf{M}_1\boldsymbol{\beta}_1 + \mathbf{W}_1\mathbf{M}_1\boldsymbol{\alpha}_1 + \mathbf{B}^T_{21}\boldsymbol{\gamma}_2$$

$$\tau_n = \mathbf{p}_n^T\boldsymbol{\gamma}_n$$
$$\tau_{n-1} = \mathbf{p}_{n-1}^T\boldsymbol{\gamma}_{n-1}$$
$$\vdots$$
$$\tau_1 = \mathbf{p}_1^T\boldsymbol{\gamma}_1$$



$$\boldsymbol{\alpha}_1 = \mathbf{p}_1\dot{\theta}_1$$
$$\boldsymbol{\alpha}_2 = \mathbf{p}_2\dot{\theta}_2 + \boldsymbol{\alpha}_1$$
$$\vdots$$
$$\boldsymbol{\alpha}_n = \mathbf{p}_n\dot{\theta}_n + \boldsymbol{\alpha}_{n-1}$$

$$\boldsymbol{\beta}_1 = \mathbf{p}_1\ddot{\theta}_1 + \boldsymbol{\Omega}_1\mathbf{p}_1\dot{\theta}_1$$
$$\boldsymbol{\beta}_2 = \mathbf{p}_2\ddot{\theta}_2 + \boldsymbol{\Omega}_2\mathbf{p}_2\dot{\theta}_2 + \mathbf{B}_{21}\boldsymbol{\beta}_1 + \dot{\mathbf{B}}_{21}\boldsymbol{\alpha}_1$$
$$\vdots$$
$$\boldsymbol{\beta}_n = \mathbf{p}_n\ddot{\theta}_n + \boldsymbol{\Omega}_n\mathbf{p}_n\dot{\theta}_n + \mathbf{B}_{n,n-1}\boldsymbol{\beta}_{n-1} + \dot{\mathbf{B}}_{n,n-1}\boldsymbol{\alpha}_{n-1}$$

Recursive Forward Dynamics & Simulation

1999 IJRR, V. 18, N. 1, pp. 20-36

$$\boxed{\mathbf{I}\ddot{\theta} + \mathbf{C}\dot{\theta} = \tau}$$

$$\mathbf{I} \equiv \mathbf{UDU}^T$$

$$\phi \equiv \tau - \mathbf{C}\dot{\theta}$$

$$\boxed{\mathbf{UDU}^T\ddot{\theta} = \phi}$$

$$\hat{\tau} = \mathbf{U}^{-1}\phi$$

$$\tilde{\tau}_i = \hat{\tau}_i / \hat{m}_i$$

$$\ddot{\theta} = \mathbf{U}^{-T}\tilde{\tau}$$

$$\hat{\tau}_i = \phi_i - \mathbf{p}_i^T \eta_{i,i+1} \quad \eta_{i,i+1} \equiv \mathbf{B}_{i+1,i}^T \eta_{i+1} \quad \eta_{i+1} \equiv \psi_{i+1}\hat{\tau}_{i+1} + \eta_{i+1,i+2} \quad \hat{\tau}_n \equiv \phi_n$$

$$\ddot{\theta}_i = \tilde{\tau}_i - \psi_i^T \mu_{i,i-1} \quad \mu_{i-1} \equiv \mathbf{p}_{i-1}\ddot{\theta}_{i-1} + \mu_{i-1,i-2} \quad \mu_{i,i-1} \equiv \mathbf{B}_{i,i-1}\mu_{i-1} \quad \mu_{1,0} = \mathbf{0}$$

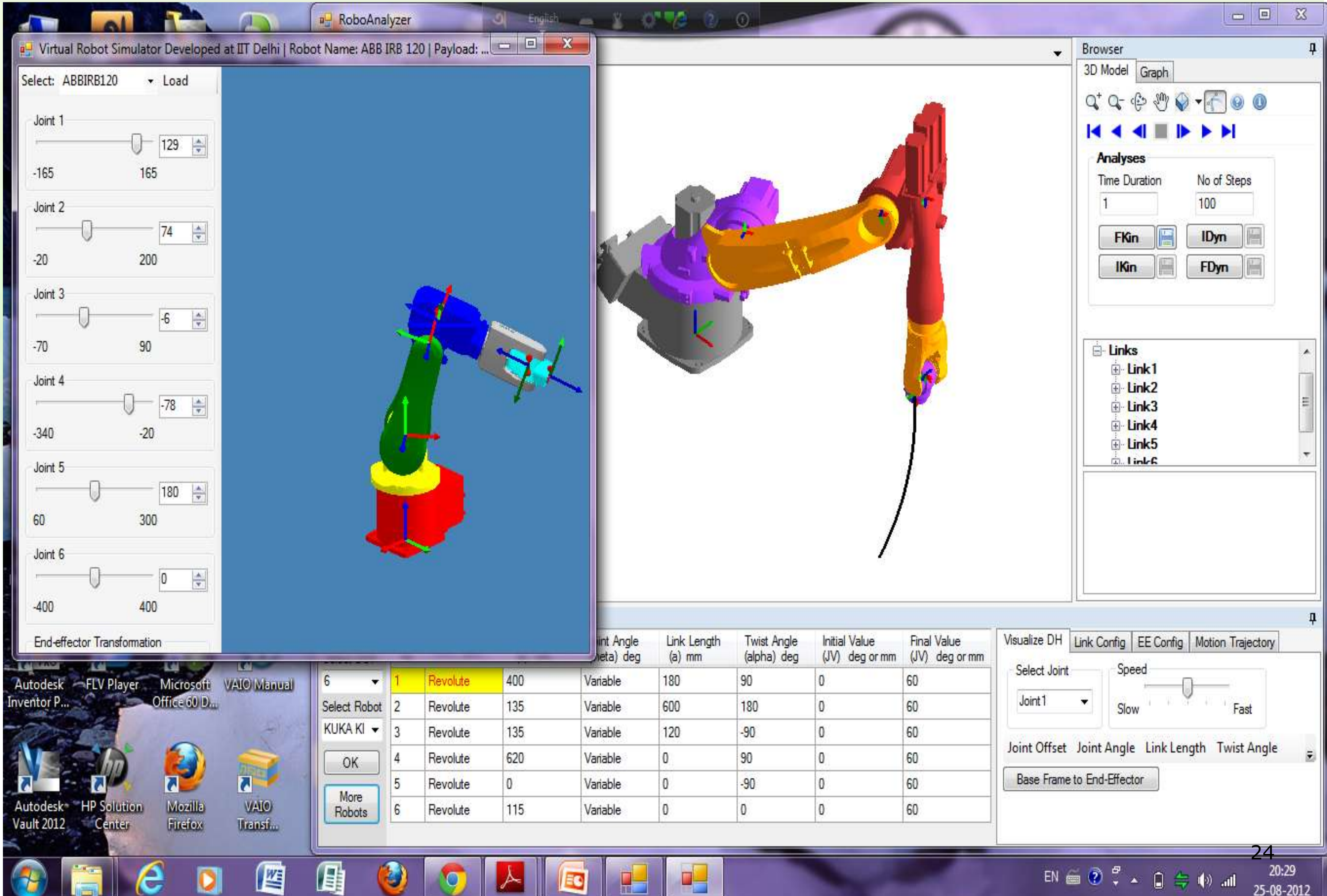
$$\hat{\psi}_k \equiv \hat{\mathbf{M}}_k \mathbf{p}_k, \qquad \hat{\psi}_{ik} \equiv \mathbf{B}_{ki}^T \hat{\psi}_k, \qquad \hat{m}_k \equiv \mathbf{p}_k^T \hat{\psi}_k,$$

$$\psi_k \equiv \frac{\hat{\psi}_k}{\hat{m}_k}, \quad \text{and} \quad \psi_{ik} \equiv \frac{\hat{\psi}_{ik}}{\hat{m}_k}$$

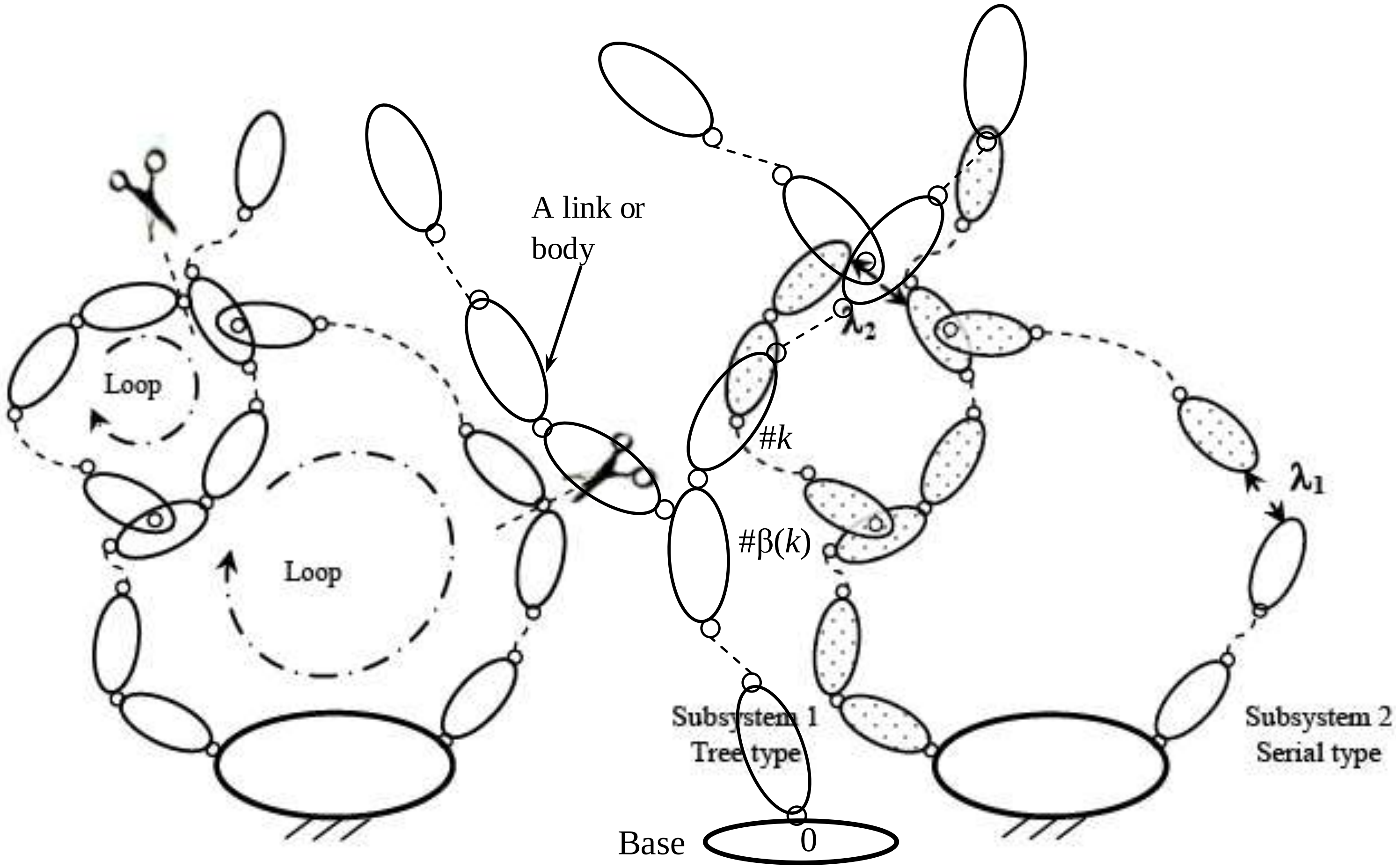
Articulated body matrix

$$\boxed{\hat{\mathbf{M}}_{ik} = \mathbf{M}_i + \mathbf{B}_{i+1,i}^T \hat{\mathbf{M}}_{i+1,k} \mathbf{B}_{i+1,i}}$$

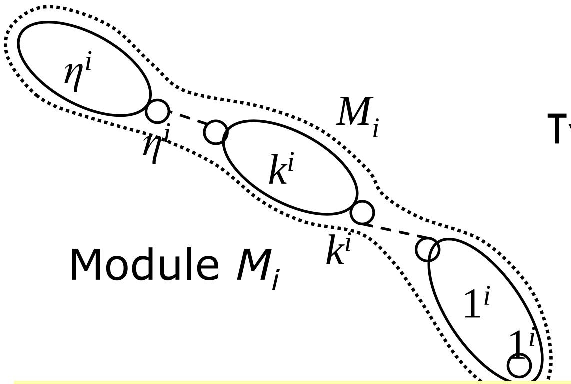
Books + RoboAnalyzer (Free: www.roboanalyzer.com)



Tree-type and Closed Systems



Intra-modular Constraints (Inside the module)

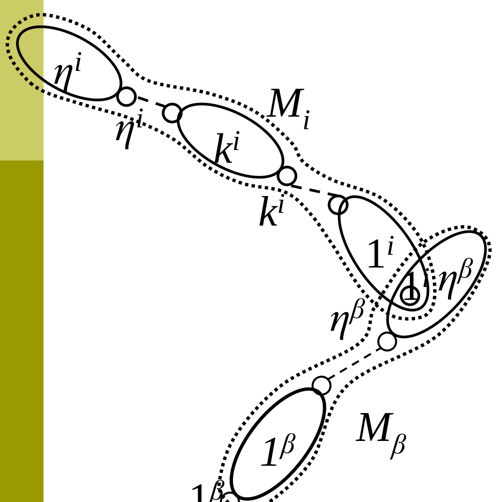


Module M_i

Twist of the k^{th} link $\quad \mathbf{t}_k = \begin{bmatrix} \boldsymbol{\omega}_k \\ \dot{\mathbf{o}}_k \end{bmatrix}$

$\mathbf{t}_k$ terms of $\mathbf{t}_{k\text{-}1}$ $\quad \mathbf{t}_k = \mathbf{A}_{k,k-1}\mathbf{t}_{k-1} + \mathbf{p}_k\dot{\theta}_k$

Inter-modular Constraints (Between the modules)



Module M_i and M_β

Module twist and module joint rate

$$\bar{\mathbf{t}}_i \equiv \begin{bmatrix} \mathbf{t}_1 \\ \vdots \\ \mathbf{t}_k \\ \vdots \\ \mathbf{t}_\eta \end{bmatrix}^i \text{ and } \dot{\bar{\boldsymbol{\theta}}}_i \equiv \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_k \\ \vdots \\ \dot{\theta}_\eta \end{bmatrix}^i$$

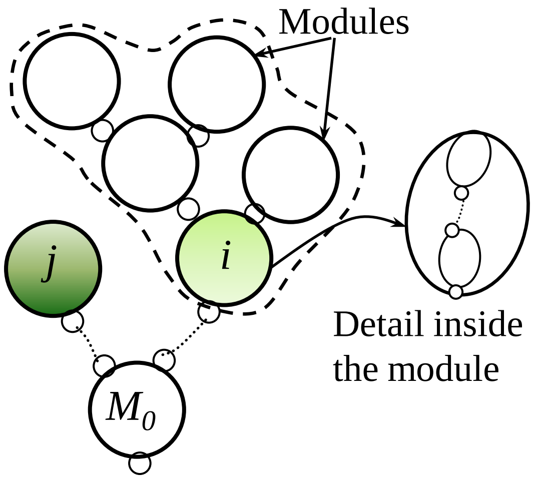
Module twist $\bar{\mathbf{t}}_i$ in terms of $\bar{\mathbf{t}}_\beta$

$$\bar{\mathbf{t}}_i = \bar{\mathbf{A}}_{i,\beta}\bar{\mathbf{t}}_\beta + \bar{\mathbf{N}}_i\dot{\bar{\boldsymbol{\theta}}}_i$$

DeNOC Matrices

For (s + 1) modules generalized twist $\mathbf{t}$ and rate $\dot{\mathbf{q}}$

$$\mathbf{t} \equiv \begin{bmatrix} \bar{\mathbf{t}}_0 \\ \bar{\mathbf{t}}_1 \\ \vdots \\ \bar{\mathbf{t}}_i \\ \vdots \\ \bar{\mathbf{t}}_s \end{bmatrix}, \text{ and } \dot{\mathbf{q}} \equiv \begin{bmatrix} \dot{\bar{\boldsymbol{\theta}}}_0 \\ \dot{\bar{\boldsymbol{\theta}}}_1 \\ \vdots \\ \dot{\bar{\boldsymbol{\theta}}}_i \\ \vdots \\ \dot{\bar{\boldsymbol{\theta}}}_s \end{bmatrix}$$



Substituing $\bar{\mathbf{t}}_i$, for $i = 1, \ldots, s$,

$$\mathbf{t} = \mathbf{N}_l \mathbf{N}_d \dot{\mathbf{q}},$$

$$\mathbf{N}_l \equiv \begin{bmatrix} \bar{\mathbf{1}}_0 & & & \mathbf{O}\text{'s} & \\ \bar{\mathbf{A}}_{1,0} & \bar{\mathbf{1}}_1 & & & \\ \bar{\mathbf{A}}_{2,0} & \bar{\mathbf{A}}_{2,1} & \bar{\mathbf{1}}_2 & & \\ \vdots & \vdots & \ddots & \ddots & \\ \bar{\mathbf{A}}_{s,0} & \bar{\mathbf{A}}_{s,1} & \cdots & \bar{\mathbf{A}}_{s,s-1} & \bar{\mathbf{1}}_s \end{bmatrix}; \mathbf{N}_d \equiv \begin{bmatrix} \bar{\mathbf{N}}_0 & & \mathbf{O}\text{'s} & \\ & \bar{\mathbf{N}}_1 & & \\ & & \ddots\ \bar{\mathbf{N}}_i & \\ \mathbf{O}\text{'s} & & & \ddots\ \bar{\mathbf{N}}_s \end{bmatrix}$$

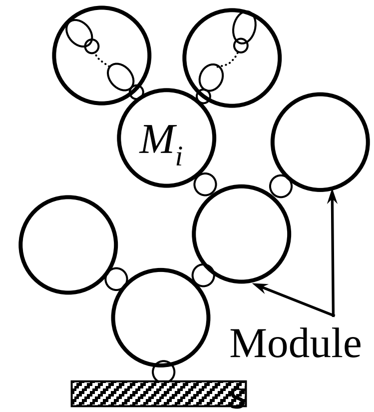
and $\bar{\mathbf{A}}_{j,i} \equiv \mathbf{O}$, if $M_j \notin \gamma_i$

$\mathbf{N}_l$ and $\mathbf{N}_d$ are the desired DeNOC matrices written in terms of module imformation

Dynamics using DeNOC matrices

NE equations for entire system

$$\mathbf{M}\dot{\mathbf{t}} + \mathbf{\Omega}\mathbf{M}\mathbf{E}\mathbf{t} = \mathbf{w}, \text{ where } \mathbf{w} = \mathbf{w}^E + \mathbf{w}^F + \mathbf{w}^C,$$



Upon pre-multiplication

$$\mathbf{N}_d^T\mathbf{N}_l^T\left(\mathbf{M}\dot{\mathbf{t}} + \mathbf{\Omega}\mathbf{M}\mathbf{E}\mathbf{t}\right) = \mathbf{N}_d^T\mathbf{N}_l^T(\mathbf{w}^E + \mathbf{w}^F), \text{ where } \mathbf{N}_d^T\mathbf{N}_l^T\mathbf{w}^C = 0$$

Equation can be written as, $\mathbf{I}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} = \boldsymbol{\tau} + \boldsymbol{\tau}^F$

Generalized inertia matrix (GIM)	$\mathbf{I} = \mathbf{N}^T\mathbf{M}\mathbf{N}$
Matrix of Convective inertia (MCI)	$\mathbf{C} = \mathbf{N}^T(\mathbf{M}\dot{\mathbf{N}} + \mathbf{W}\mathbf{M}\mathbf{E}\mathbf{N})$
Generalized external force	$\boldsymbol{\tau} = \mathbf{N}^T\mathbf{w}^E$
Generalized force other than driving (e.g. Link ground interaction)	$\boldsymbol{\tau}^F = \mathbf{N}^T\mathbf{w}^F$
and	$\mathbf{N} = \bar{\mathbf{N}}_l\bar{\mathbf{N}}_d$

Dynamics using the DeNOC Matrices

Link velocities **t** (ω and **v**) = DeNOC matrices × Rate of Independent coordinate $\dot{\theta}$

- Decou rm of the velocity transformation matrix

DeNOC matrices × Newton-Euler Equations of motion = Minimal-order Equations of motion

- Eliminate constraint forces and moments from the NE equations.

- Analytical expressions of vector and matrices, Decomposition inertia Matrix, Recursive algorithms, Dynamics model simplifications, etc.

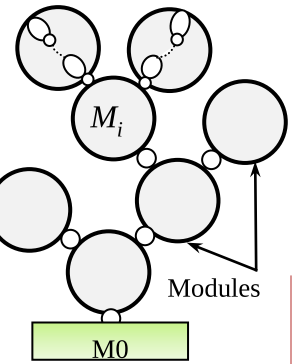
Inter- and intra- modular recursive

Forward dynamics

Inverse dynamics

Joint torques ($\bar{\boldsymbol{\tau}}_i$), inertia parameters ($\bar{\mathbf{M}}_i$), twist and motion propagations ($\bar{\mathbf{A}}_{i,j}$ and $\bar{\mathbf{N}}_i$), wrench due to foot-ground interaction ($\bar{\mathbf{w}}_i^F$) and initial conditions $\bar{\mathbf{q}}_i$ and $\dot{\bar{\mathbf{q}}}_i$

Joint motions ($\bar{\mathbf{q}}_i$, $\dot{\bar{\mathbf{q}}}_i$, $\ddot{\bar{\mathbf{q}}}_i$), inertia parameters ($\bar{\mathbf{M}}_i$), twist and motion propagations ($\bar{\mathbf{A}}_{i,\beta}$ and $\bar{\mathbf{N}}_i$), and wrench due to foot-ground interaction $\bar{\mathbf{w}}_i$



Inter-modular

Intra-modular

Forward recursion

Step 1

$i = 1:s$

$$\bar{\mathbf{t}}_i = \bar{\mathbf{A}}_{i,\beta}\bar{\mathbf{t}}_{\beta_i} + \bar{\mathbf{N}}_i\dot{\bar{\boldsymbol{\theta}}}_i$$

$$\dot{\bar{\mathbf{t}}}_i = \bar{\mathbf{A}}_{i,\beta}\dot{\bar{\mathbf{t}}}_{\beta_i} + \dot{\bar{\mathbf{A}}}_{i,\beta}\bar{\mathbf{t}}_{\beta_i} + \dot{\bar{\mathbf{N}}}_i\dot{\bar{\boldsymbol{\theta}}}_i$$

$$\bar{\mathbf{w}}_i^* = \bar{\mathbf{M}}_i\dot{\bar{\mathbf{t}}}_i + \bar{\boldsymbol{\Omega}}_i\bar{\mathbf{M}}_i\bar{\mathbf{E}}_i\bar{\mathbf{t}}_i$$

$i = 1:s$

$$\bar{\mathbf{t}}_i = \bar{\mathbf{A}}_{i,\beta}\bar{\mathbf{t}}_{\beta_i} + \bar{\mathbf{N}}_i\dot{\bar{\boldsymbol{\theta}}}_i$$

$$\dot{\bar{\mathbf{t}}}_i = \dot{\bar{\mathbf{A}}}_{i,\beta}\bar{\mathbf{t}}_{\beta_i} + \bar{\mathbf{A}}_{i,\beta}\dot{\bar{\mathbf{t}}}_{\beta_i} + \dot{\bar{\mathbf{N}}}_i\dot{\bar{\boldsymbol{\theta}}}_i + \bar{\mathbf{N}}_i\ddot{\bar{\boldsymbol{\theta}}}_i$$

$$\tilde{\bar{\mathbf{w}}}_i = \bar{\mathbf{M}} \ldots$$

$k = 1:\eta^i,\ r = 1:\varepsilon_k$

$$\mathbf{t}_j = \mathbf{A}_{k,\beta}\mathbf{t}_{\beta(j)} + \mathbf{p}_j\dot{\theta}_j,\ r = 1$$

$$= \mathbf{p}_{k_j}\dot{\theta}_{k_j},\ r > 1$$

$$\dot{\mathbf{t}}_j = \mathbf{A}_{k,\beta}\dot{\mathbf{t}}_{\beta(j)} + \dot{\mathbf{A}}_{k,\beta}\mathbf{t}_{\beta(j)} + \boldsymbol{\Omega}_j\mathbf{p}_j\dot{\theta}_j,\ r = 1$$

$$= \boldsymbol{\Omega}_j\mathbf{p}_j\dot{\theta}_j,\ r > 1$$

$$\tilde{\mathbf{w}}_j = 0,\ r < \varepsilon_k$$

$$= \mathbf{M}_k\dot{\mathbf{t}}_j + \boldsymbol{\Omega}_j\mathbf{M}_k\mathbf{E}_k\mathbf{t}_j,\ r = \varepsilon_k$$

$k = 1:\eta^i$

$$\mathbf{t}_j = \mathbf{A}_{k,\beta} \ldots$$

$$\dot{\mathbf{t}}_j = \mathbf{A}_{k,\beta}\dot{\mathbf{t}}_{\beta(j)} + \mathbf{A}_{k,\beta}\mathbf{t}_{\beta(j)} + \boldsymbol{\Omega}_j\mathbf{p}_j\dot{\theta}_j + \mathbf{p}_j\ddot{\theta}_j,\ r = 1$$

$$= \boldsymbol{\Omega}_j\mathbf{p}_j \ldots$$

$$\tilde{\mathbf{w}}_j = 0, \ldots$$

$$= \mathbf{M} \ldots$$

Step 2

$i = s:1$

$$\hat{\bar{\boldsymbol{\Psi}}}^i = \hat{\bar{\mathbf{M}}}^i\bar{\mathbf{N}}^i;\ \hat{\bar{\mathbf{I}}}_i = \bar{\mathbf{N}}_i^T\hat{\bar{\boldsymbol{\Psi}}}_i;\ \bar{\boldsymbol{\Psi}}_i = \hat{\bar{\boldsymbol{\Psi}}}_i\hat{\bar{\mathbf{I}}}_i^{-1}$$

$$\hat{\bar{\boldsymbol{\varphi}}}_i = \bar{\boldsymbol{\varphi}}_i - \bar{\mathbf{N}}_i^T\tilde{\bar{\boldsymbol{\eta}}}_i$$

$$\tilde{\bar{\boldsymbol{\varphi}}}_i = \hat{\bar{\mathbf{I}}}_i^{-1}\hat{\bar{\boldsymbol{\varphi}}}_i$$

$$\hat{\bar{\mathbf{M}}}_{i,i} = \hat{\bar{\mathbf{M}}}_i - \hat{\bar{\boldsymbol{\Psi}}}_i\bar{\boldsymbol{\Psi}}_i^T;\ \bar{\boldsymbol{\eta}}_i = \bar{\boldsymbol{\Psi}}_i\hat{\bar{\boldsymbol{\varphi}}}_i + \tilde{\bar{\boldsymbol{\eta}}}_i$$

$$\hat{\bar{\mathbf{M}}}_{\beta(i)} = \hat{\bar{\mathbf{M}}}_{\beta(i)} + \bar{\mathbf{A}}_{i,\beta(i)}^T\hat{\bar{\mathbf{M}}}_{i,i}\bar{\mathbf{A}}_{i,\beta(i)}$$

$$\tilde{\bar{\boldsymbol{\eta}}}_{\beta(i)} = \tilde{\bar{\boldsymbol{\eta}}}_{\beta(i)} + \bar{\mathbf{A}}_{i,\beta(i)}^T\bar{\boldsymbol{\eta}}_i$$

$k = \eta^i:1,\ r = \varepsilon_k:1$

$$\hat{\boldsymbol{\psi}}_j = \hat{\mathbf{M}}_j\mathbf{p}_j;\ \hat{m}_j = \mathbf{p}_j^T\hat{\boldsymbol{\psi}}_j;\ \boldsymbol{\psi}_j = \hat{\boldsymbol{\psi}}_j/\hat{m}_j$$

$$\hat{\varphi}_j = \varphi_j - \mathbf{p}_j^T\tilde{\boldsymbol{\eta}}_j$$

$$\tilde{\varphi}_j = \hat{\varphi}_j/\hat{m}_j$$

$$\hat{\mathbf{M}}_{j,j} = \hat{\mathbf{M}}_j - \hat{\boldsymbol{\psi}}_j\boldsymbol{\psi}_j^T;\ \boldsymbol{\eta}_j = \boldsymbol{\psi}_j\hat{\varphi}_j + \tilde{\boldsymbol{\eta}}_j$$

$$\hat{\mathbf{M}}_{\beta(j)} = \hat{\mathbf{M}}_{\beta(j)} + \mathbf{A}_{k,\beta}^T\hat{\mathbf{M}}_{j,j}\mathbf{A}_{k,\beta},\ r > 1$$

$$= \hat{\mathbf{M}}_{j,j},\ r = 1$$

$$\tilde{\boldsymbol{\eta}}_{\beta(j)} = \tilde{\boldsymbol{\eta}}_{\beta(j)} + \mathbf{A}_{k,\beta}^T\boldsymbol{\eta}_j,\ r > 1$$

$$= \boldsymbol{\eta}_j,\ r = 1$$

Backward recursion

$i = \ldots$

$$\bar{\boldsymbol{\tau}}_i = \bar{\mathbf{N}}_i^T\bar{\mathbf{w}}_i$$

$$\tilde{\bar{\mathbf{w}}}_{\beta(i)} = \tilde{\bar{\mathbf{w}}}_{\beta(i)} + \bar{\mathbf{A}}_{i,\beta(i)}^T\tilde{\bar{\mathbf{w}}}_i$$

$k = \ldots$

$$\tau \ldots$$

$$\tilde{\mathbf{w}}_{\beta(j)} \ldots \mathbf{A}_{k,\beta(k)}^T\tilde{\mathbf{w}}_j,\ r = 1$$

Step 3

$i = 1:s$

$$\tilde{\bar{\boldsymbol{\mu}}}_i = \mathbf{A}^{i,\beta(i)}(\bar{\mathbf{N}}_{\beta(i)}\ddot{\bar{\mathbf{q}}}_{\beta(i)} + \tilde{\bar{\boldsymbol{\mu}}}_{\beta(i)})$$

$$\ddot{\bar{\mathbf{q}}}_i = \tilde{\bar{\boldsymbol{\varphi}}}_i - \bar{\boldsymbol{\Psi}}_i^T\tilde{\bar{\boldsymbol{\mu}}}_i,$$

$k = 1:\eta^i,\ r = 1:\varepsilon_k$

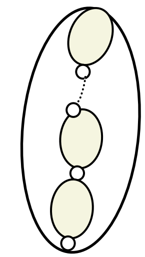
$$\tilde{\boldsymbol{\mu}}_j = \mathbf{A}_{k,\beta}(\mathbf{p}_{\beta(j)}\ddot{\theta}_{\beta(j)} + \tilde{\boldsymbol{\mu}}_{\beta(j)}),\ r > 1$$

$$= \mathbf{p}_{\beta(j)}\ddot{\theta}_{\beta(j)} + \tilde{\boldsymbol{\mu}}_{\beta(j)},\ r = 1$$

$$\ddot{\theta}_j = \tilde{\varphi}_j - \boldsymbol{\psi}_j^T\tilde{\boldsymbol{\mu}}_j$$

Joint torques ($\bar{\boldsymbol{\tau}}_i$)

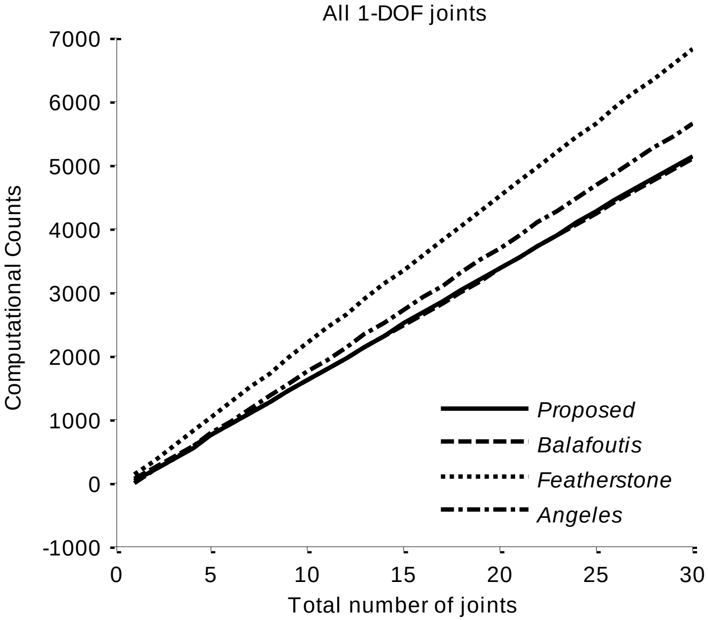
Independent accelerations $\ddot{\bar{\mathbf{q}}}_i$

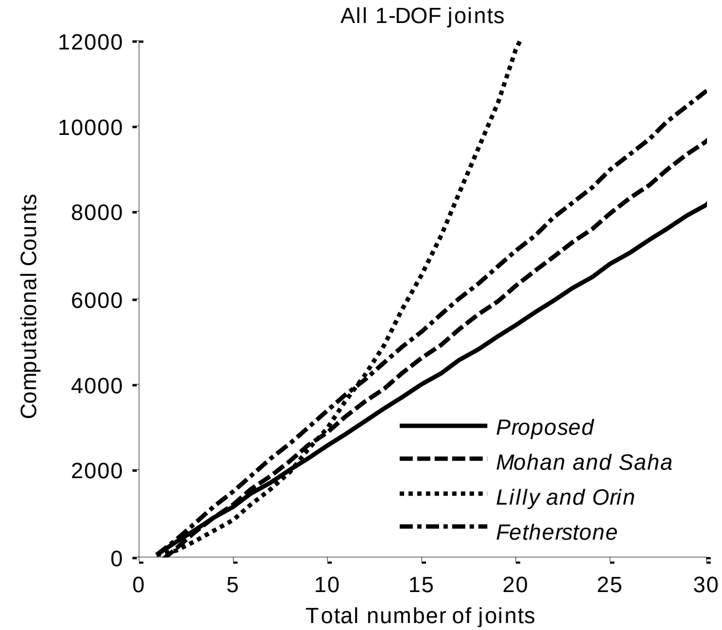


Inverse Dynamics

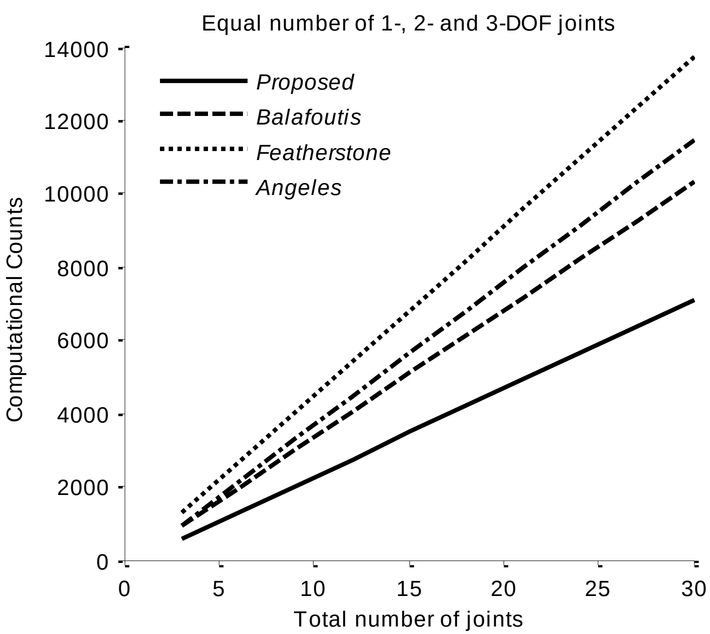
Forward Dynamics

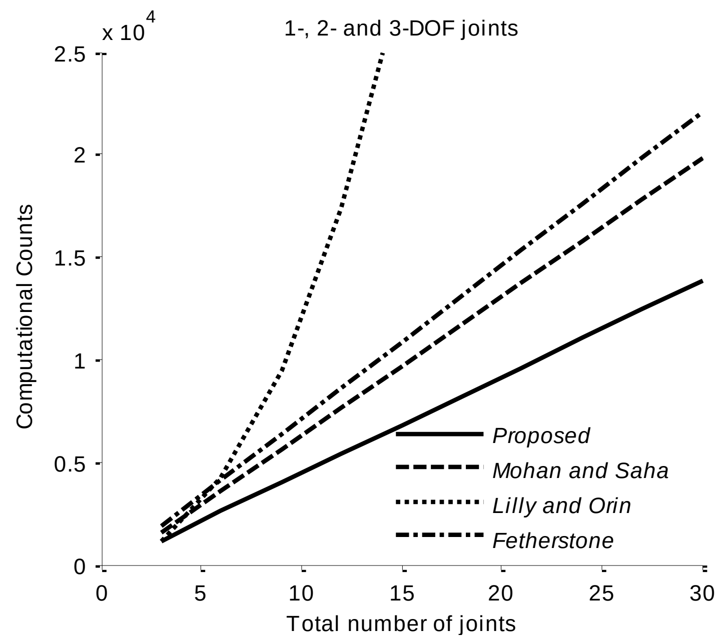
1-DOF



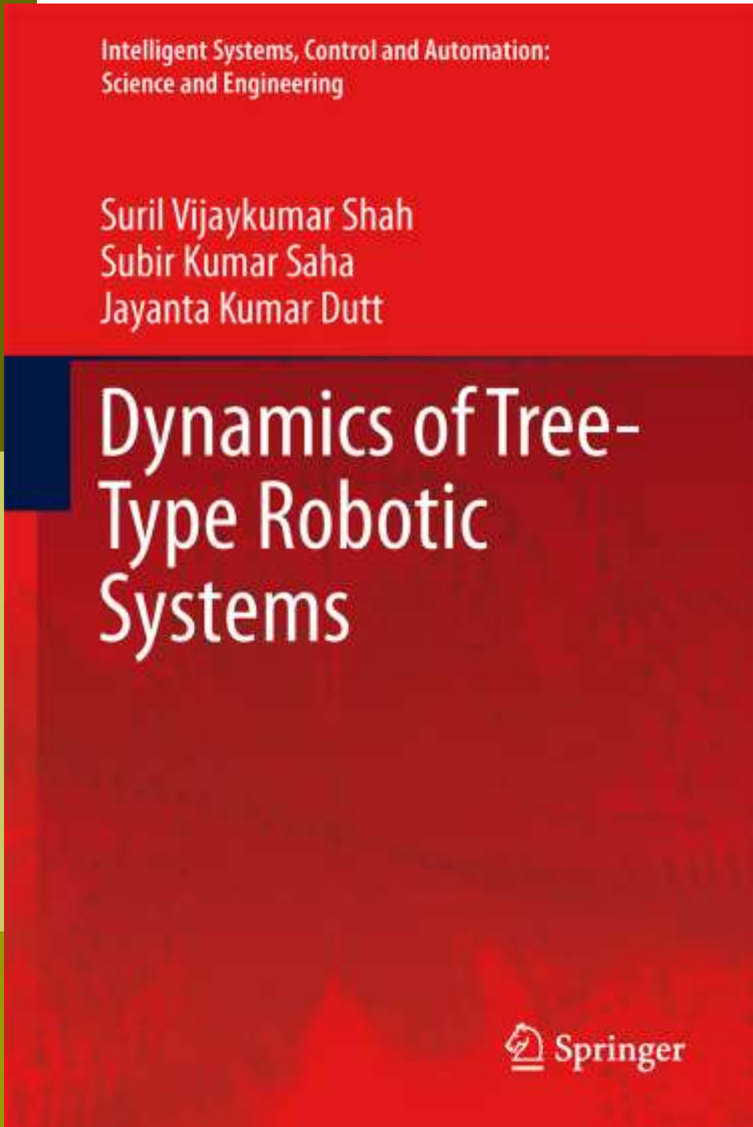


Multiple-DOF





1



2013

¥10,000

- This book addresses dynamic modelling methodology and simulation of tree-type robotic systems.
- Such analyses are required to visualize the motion of a system without really building it.
- Novel treatment of tree-type robots using

1) Kinematic modules;

2) Decoupled Natural Orthogonal Complements (DeNOC)

- Unified representation of the multiple-degrees-of freedom-joints
- Efficient <u>Recursive Dynamics Algorithms</u>
- Examples: Biped, Quadruped. Hexapod, etc.

ReDySim?

- Recursive Dynamic Simulator is Free

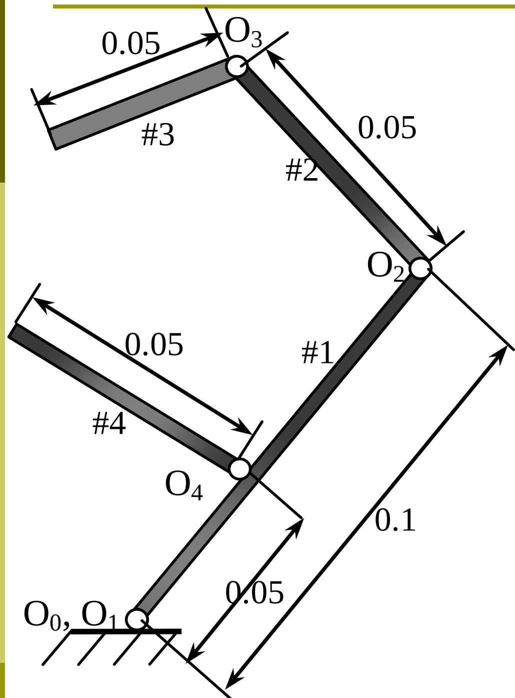
  http://www.roboanalyzer.com

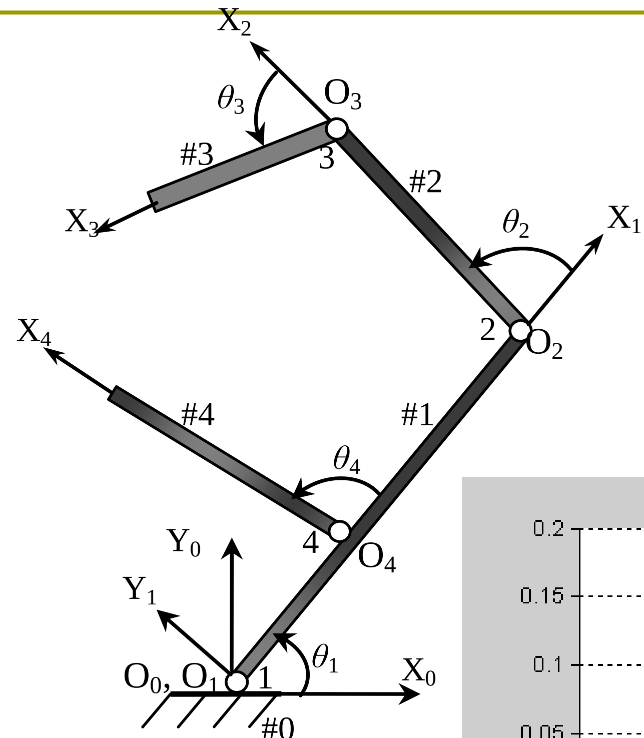
  or

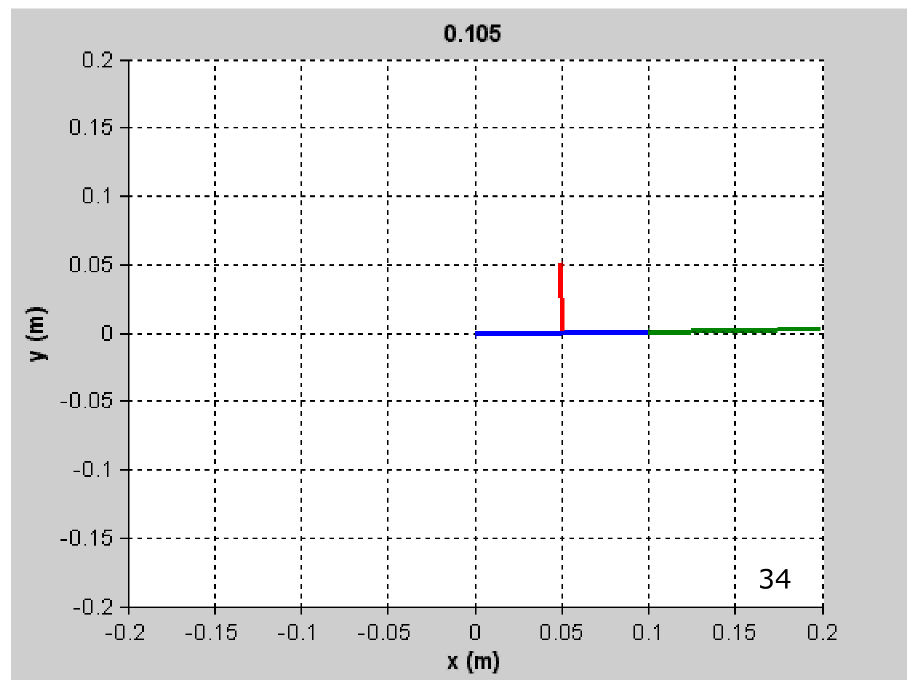
  http://www.redysim.co.nr/download

- It has three modules
  1. Open- and closed-loop multi-body systems
  2. Free-floating system
  3. Legged robot with ground interactions

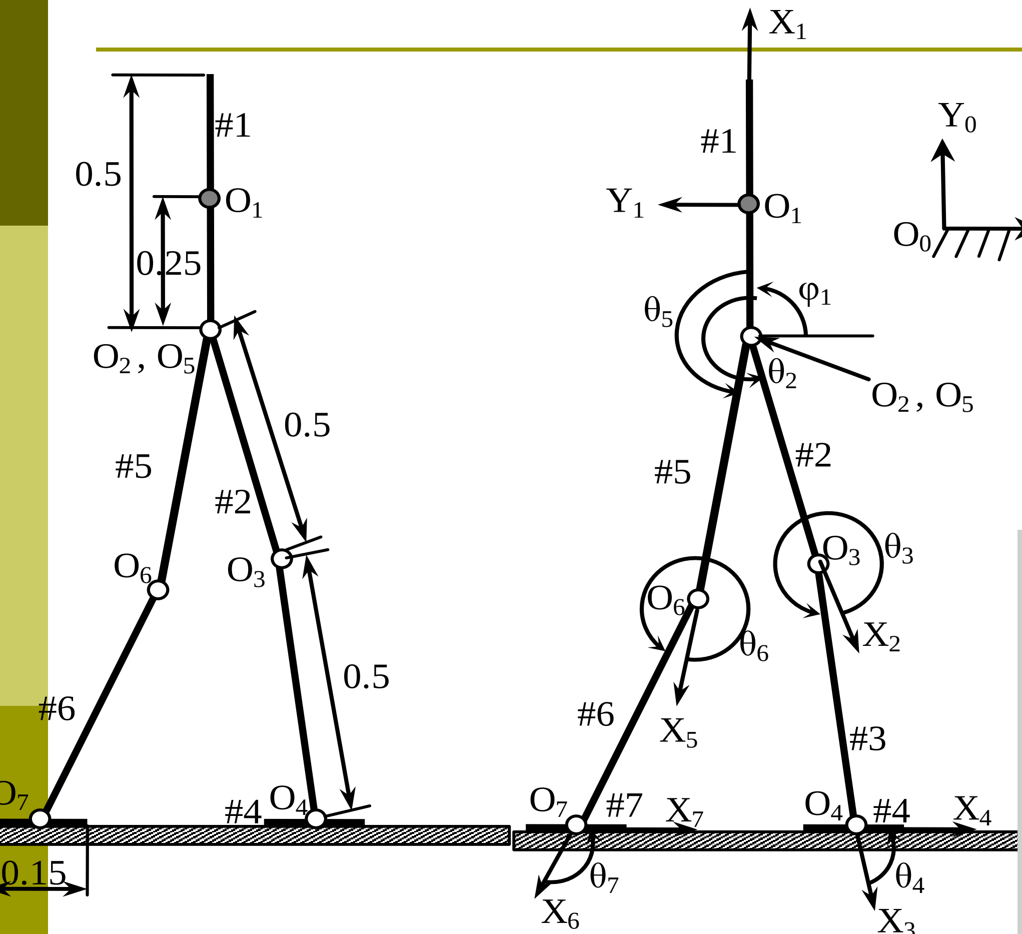
A Robotic Gripper (Inverse Dynamics)

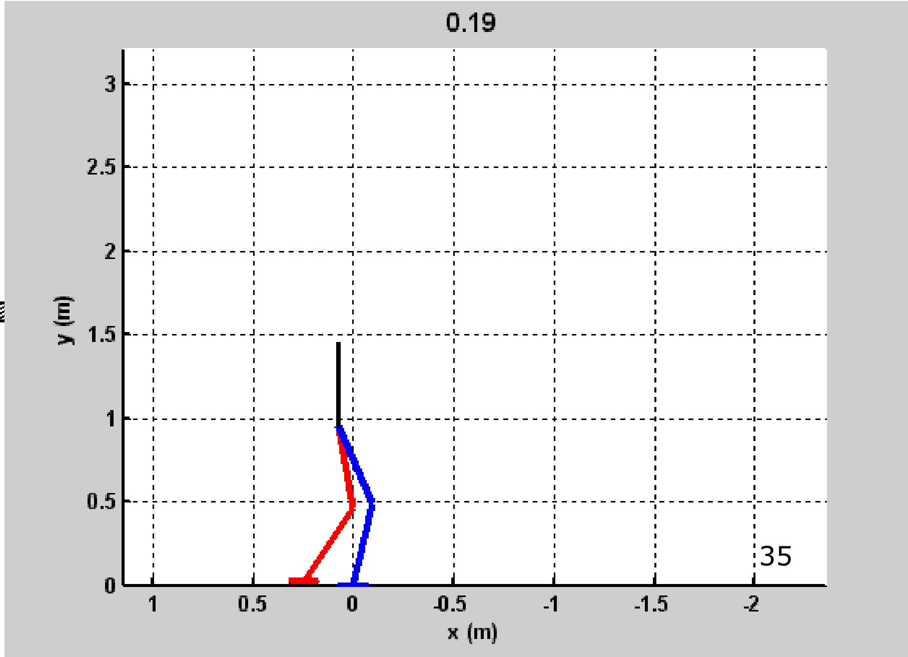




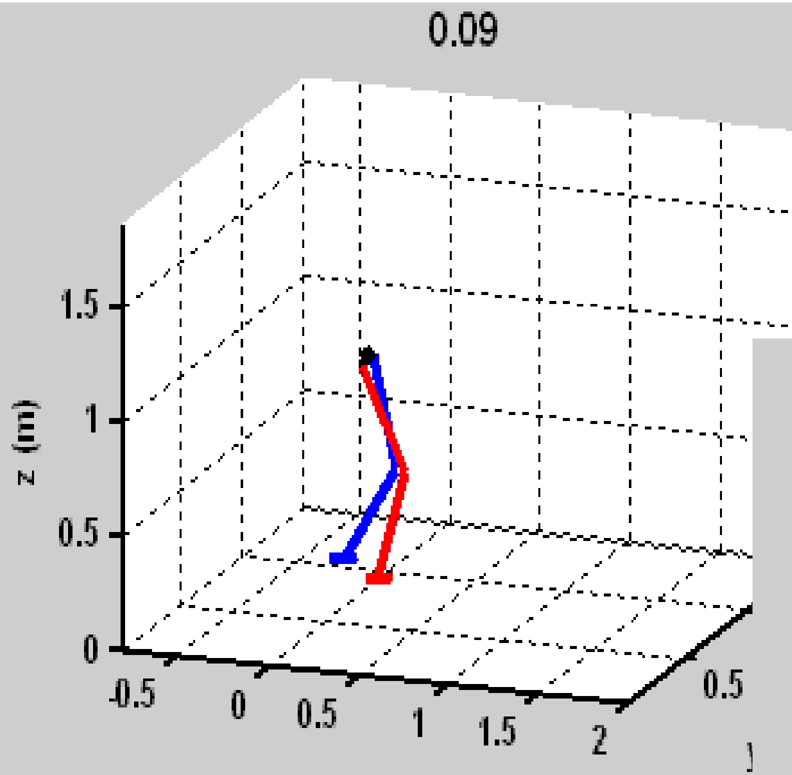


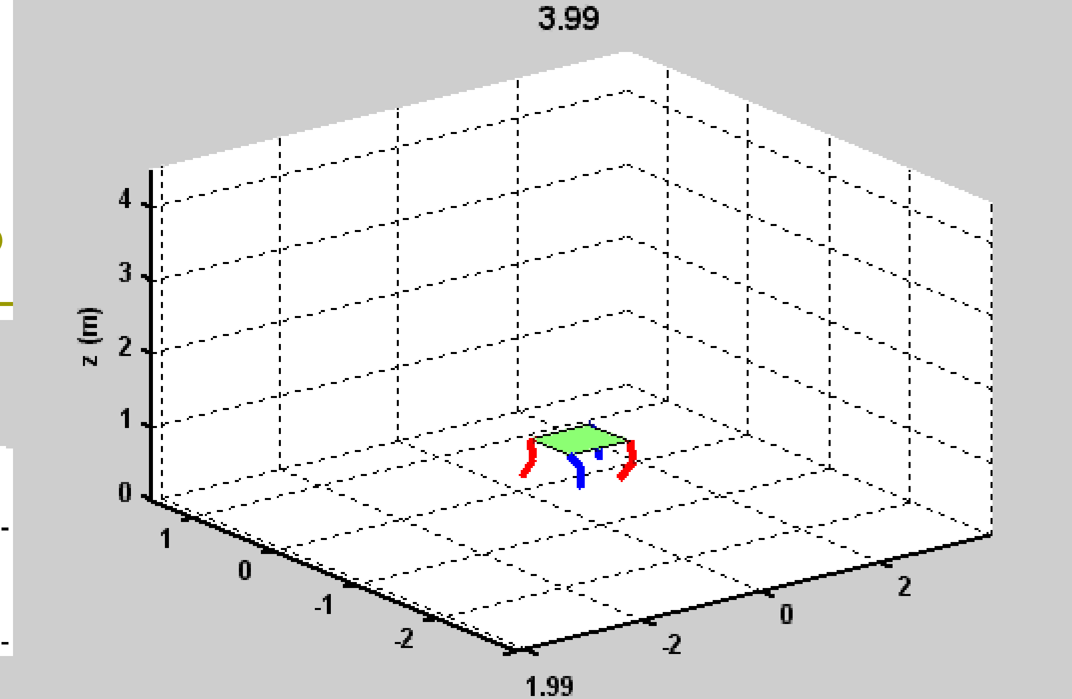
Planar biped

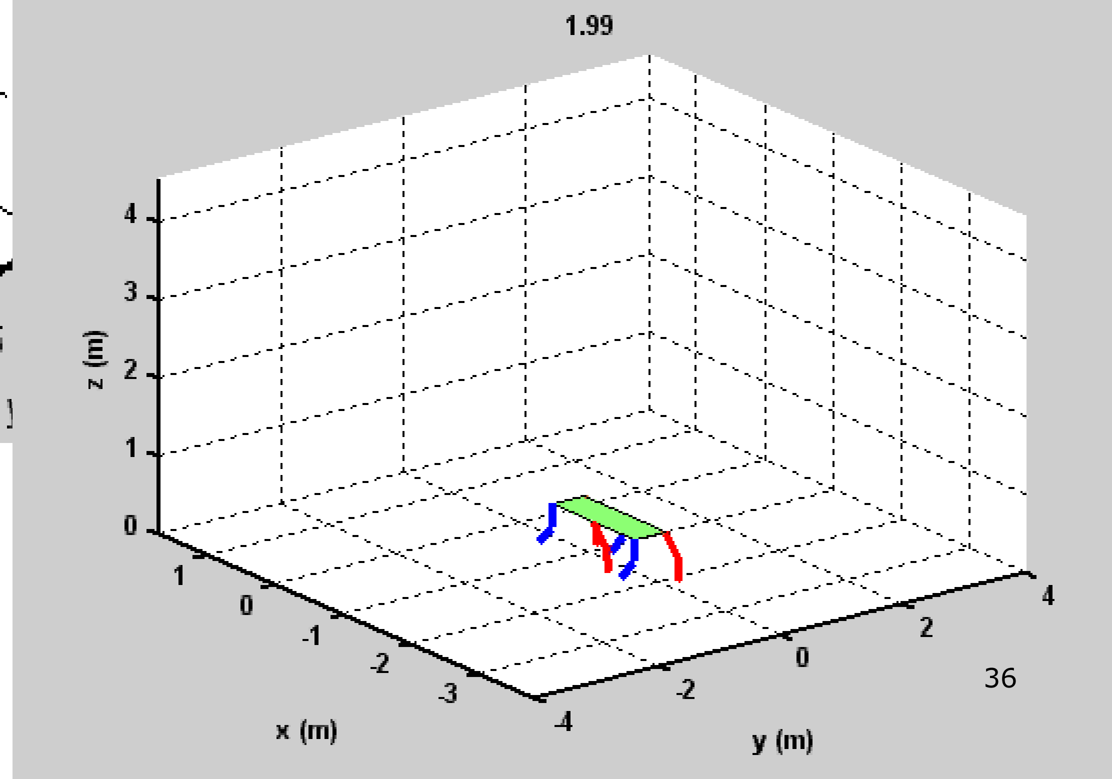




Legged robots

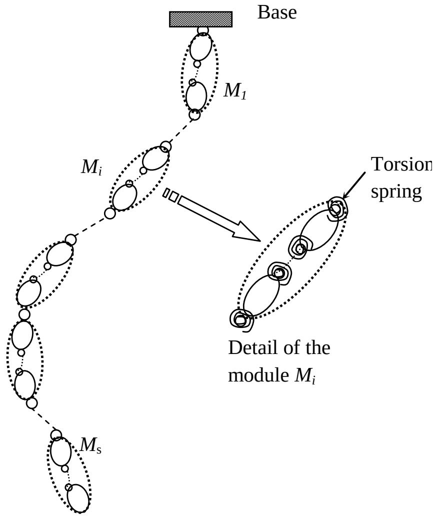


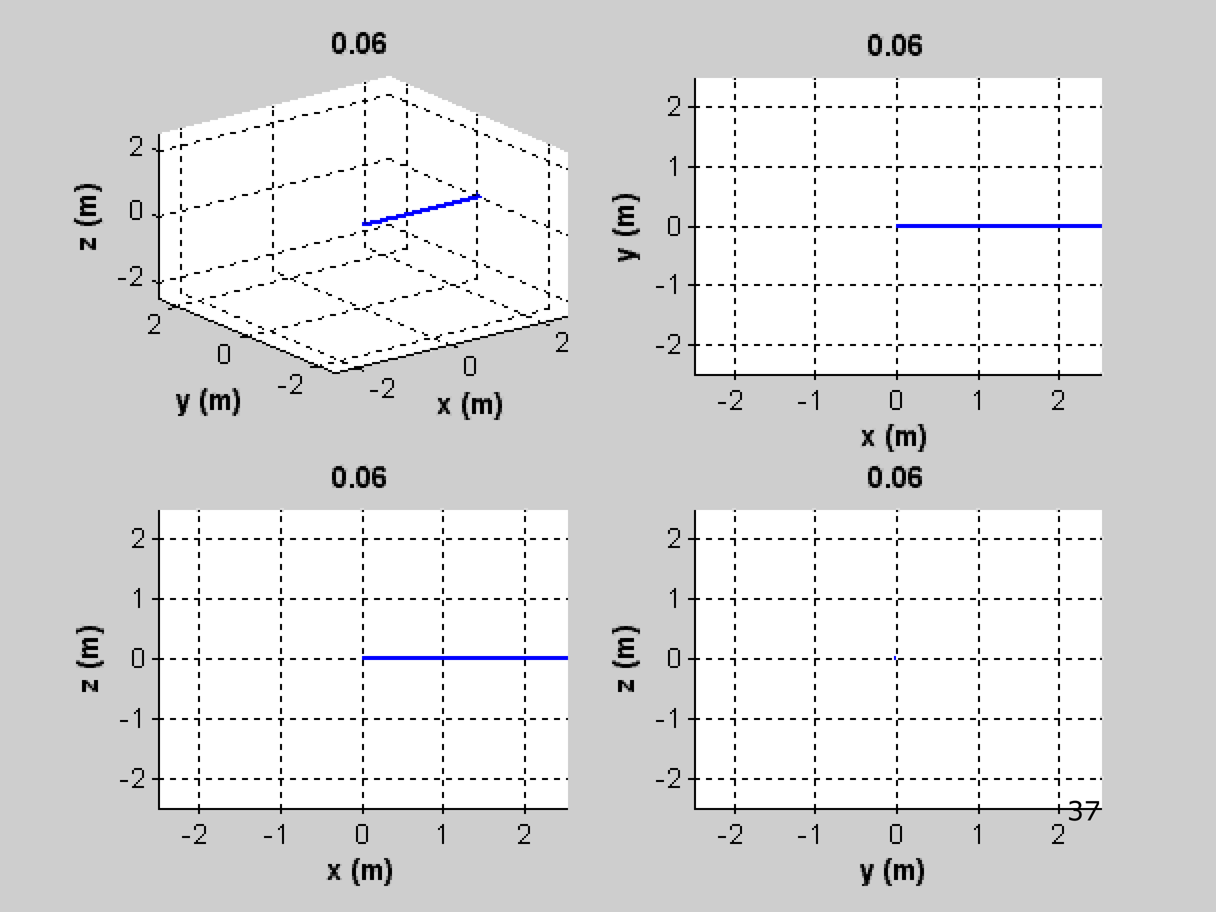




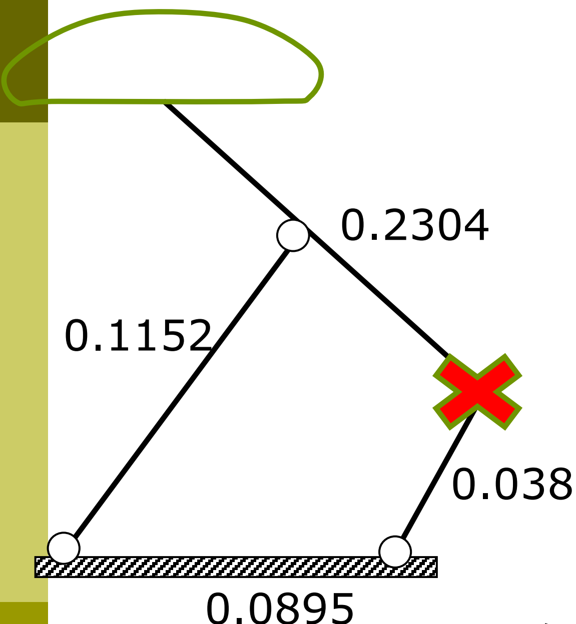
New Application: Chains and Ropes

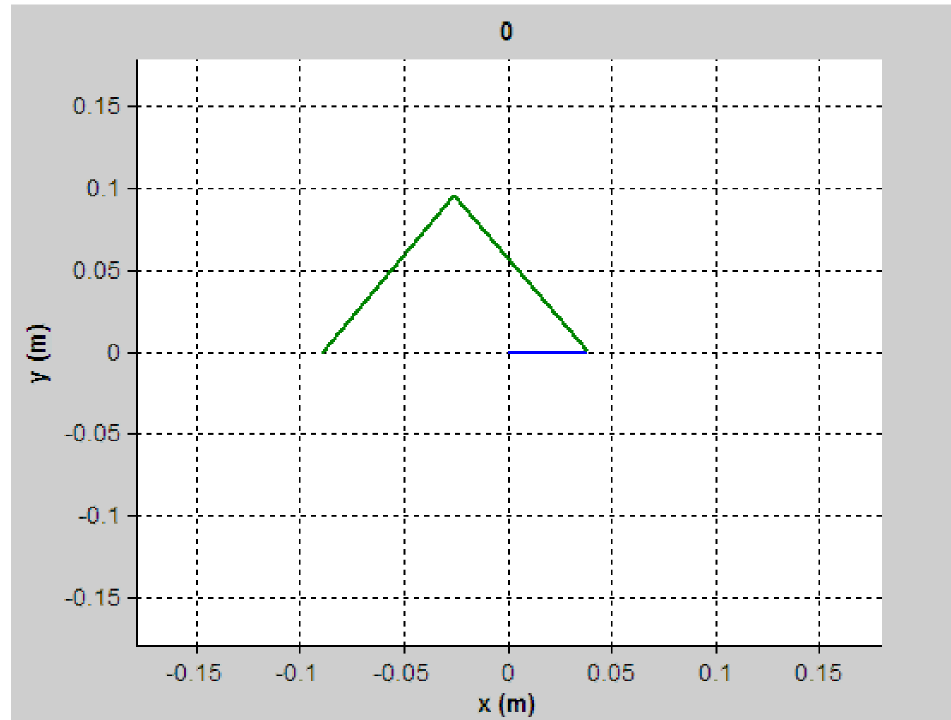
Agrawal (2013): MUB

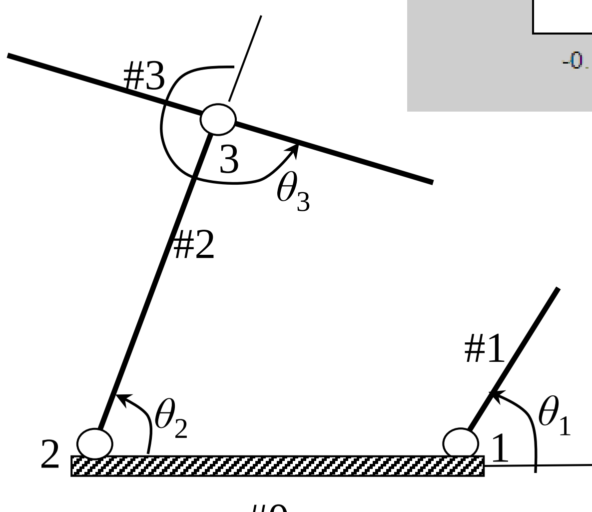




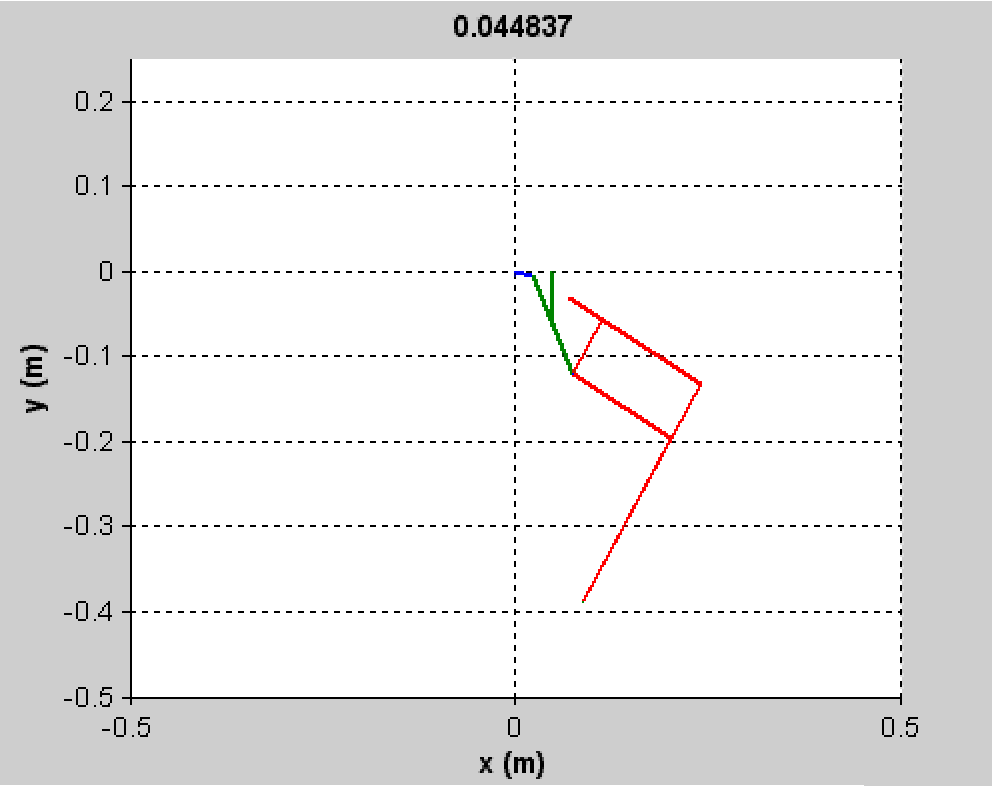
Four-bar mechanism (Forward Dynamics)

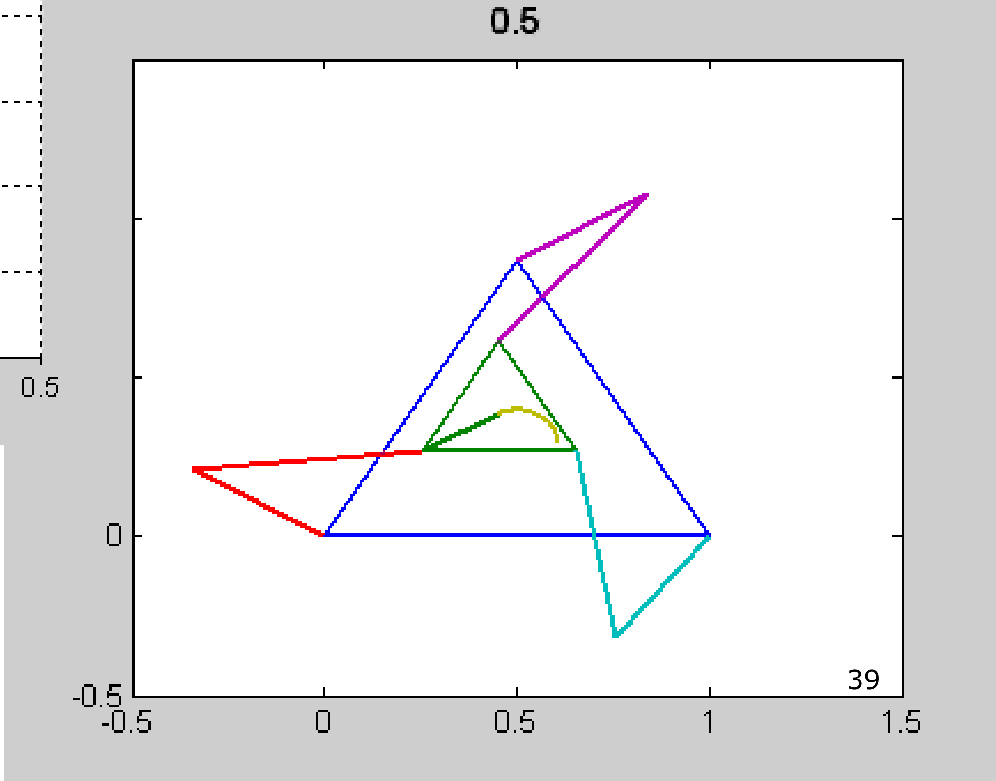






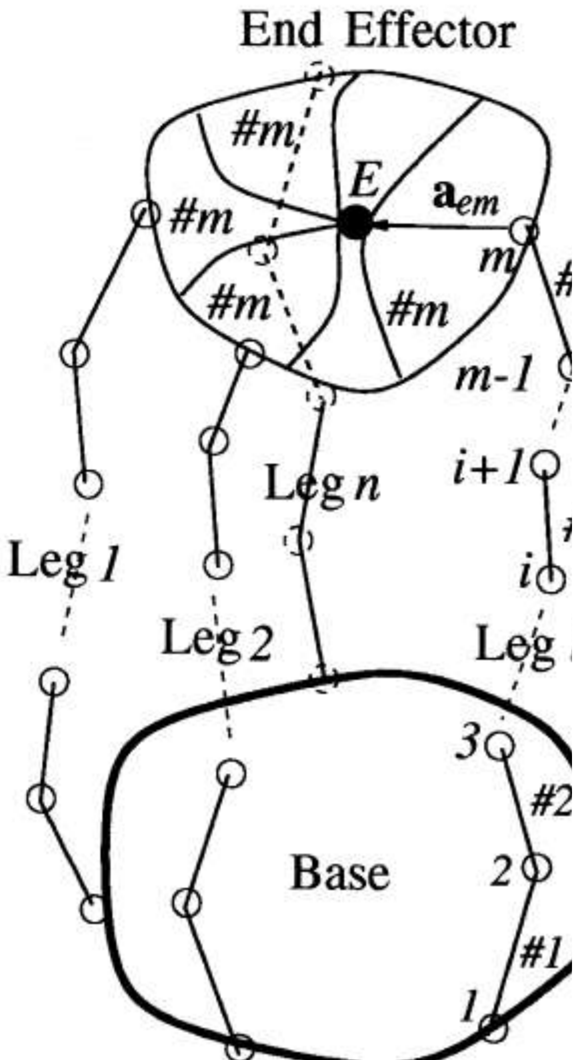
Closed-loop systems

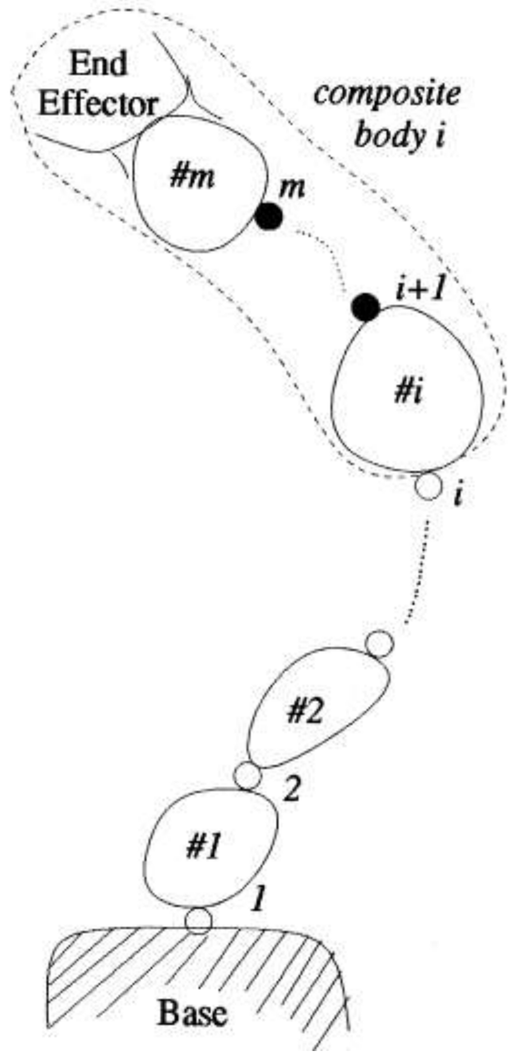




Recursive Dynamics for Closed-loops (Without Cut-method)

Courtesy: Prof. Shimizu





Saha (2001): MSM

Khan (2005): ASME MUB

Solve for Unactuated Joint-rates

$$\mathbf{t}'_e \equiv \mathbf{t}_e - \mathbf{A}_{ek}\mathbf{p}_k\dot{\theta}_k$$

$$\mathbf{t}_1 = \mathbf{p}_1\dot{\theta}_1 = \frac{1}{\delta_1}\mathbf{p}_1\tilde{\mathbf{p}}_1^T\mathbf{t}'_e$$

$$\mathbf{t}_2 = \mathbf{A}_{21}\mathbf{t}_1 + \mathbf{p}_2\dot{\theta}_2 = \boldsymbol{\Psi}_2\mathbf{A}_{21}\mathbf{t}_1 + \frac{1}{\delta_2}\mathbf{p}_2\tilde{\mathbf{p}}_2^T\mathbf{t}'_e$$

$$\vdots$$

$$\mathbf{t}_k = \mathbf{A}_{k,k-1}\mathbf{t}_{k-1} + \mathbf{p}_k\dot{\theta}_k, \qquad \text{i.e.,} \quad \boldsymbol{\Psi}_k = \mathbf{1}$$

$$\mathbf{t}_{k+1} = \mathbf{A}_{k+1,k}\mathbf{t}_k + \mathbf{p}_{k+1}\dot{\theta}_{k+1} = \boldsymbol{\Psi}_{k+1}\mathbf{A}_{k+1,k}\mathbf{t}_k + \frac{1}{\delta_{k+1}}\mathbf{p}_{k+1}\tilde{\mathbf{p}}_{k+1}^T\mathbf{t}_e$$

$$\vdots$$

$$\mathbf{t}_m = \mathbf{A}_{m,m-1}\mathbf{t}_{m-1} + \mathbf{p}_m\dot{\theta}_m = \boldsymbol{\Psi}_m\mathbf{A}_{m,m-1}\mathbf{t}_{m-1} + \frac{1}{\delta_m}\mathbf{p}_m\tilde{\mathbf{p}}_m^T\mathbf{t}_e$$

DeNOC: $\boxed{\mathbf{t} = \mathbf{N}_l\boxed{\mathbf{N}_c}\mathbf{N}_d\dot{\boldsymbol{\theta}}}$

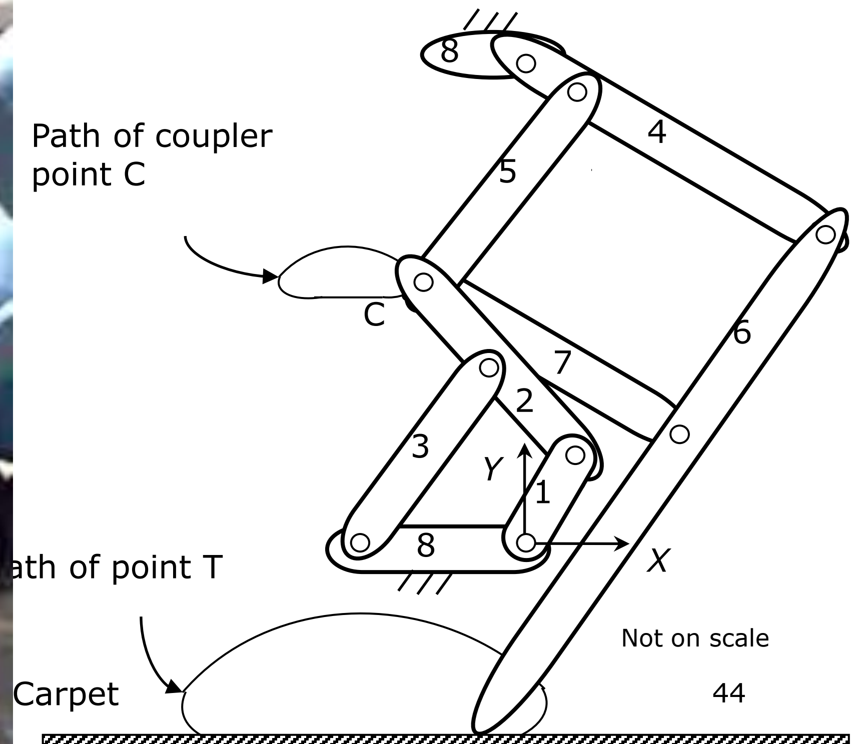
Use in MuDRA

- Multibody Dynamics for Rural Applications (MuDRA)
  - Pose rural mechanisms as research problems
  - Use of modern tools, e.g., Multibody Dyn.
  - Use of modern software
  - Able to publish
- Benefits of MuDRA
  - Establishing a culture
  - Rural problems may get solved

Carpet Cleaning: Traditional

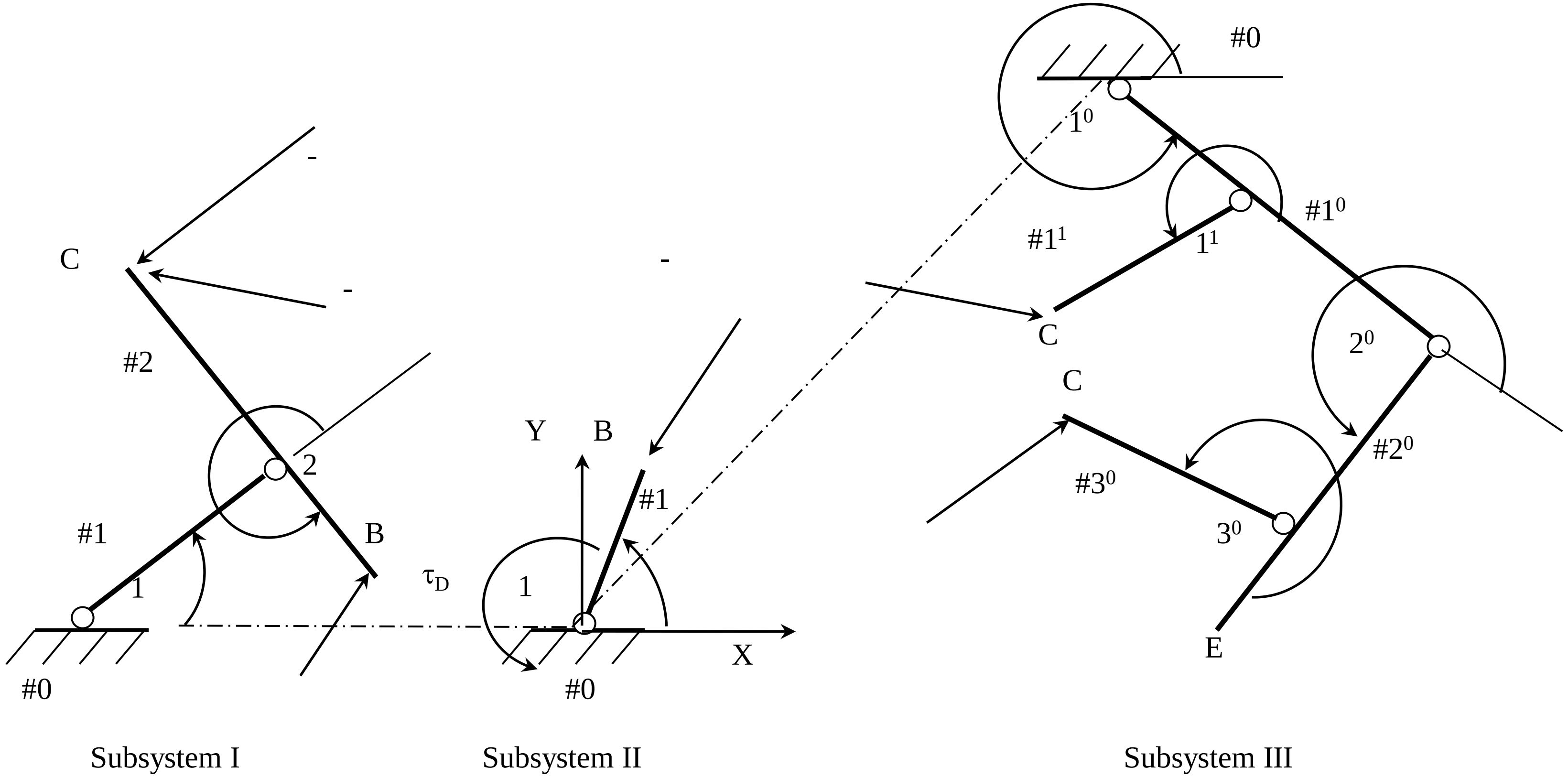
Carpet Scrapping Machine

- Purpose: To reduce human effort
- Straight line generating machine



Tree-types: Double Recursion

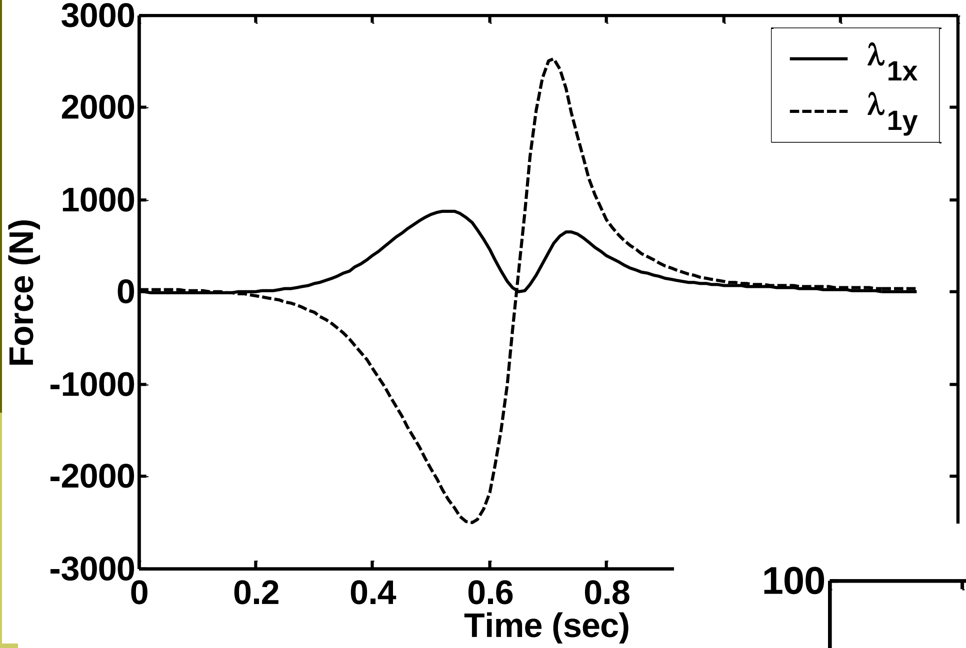
ASME J. of Mech. Des., Dec. 2007

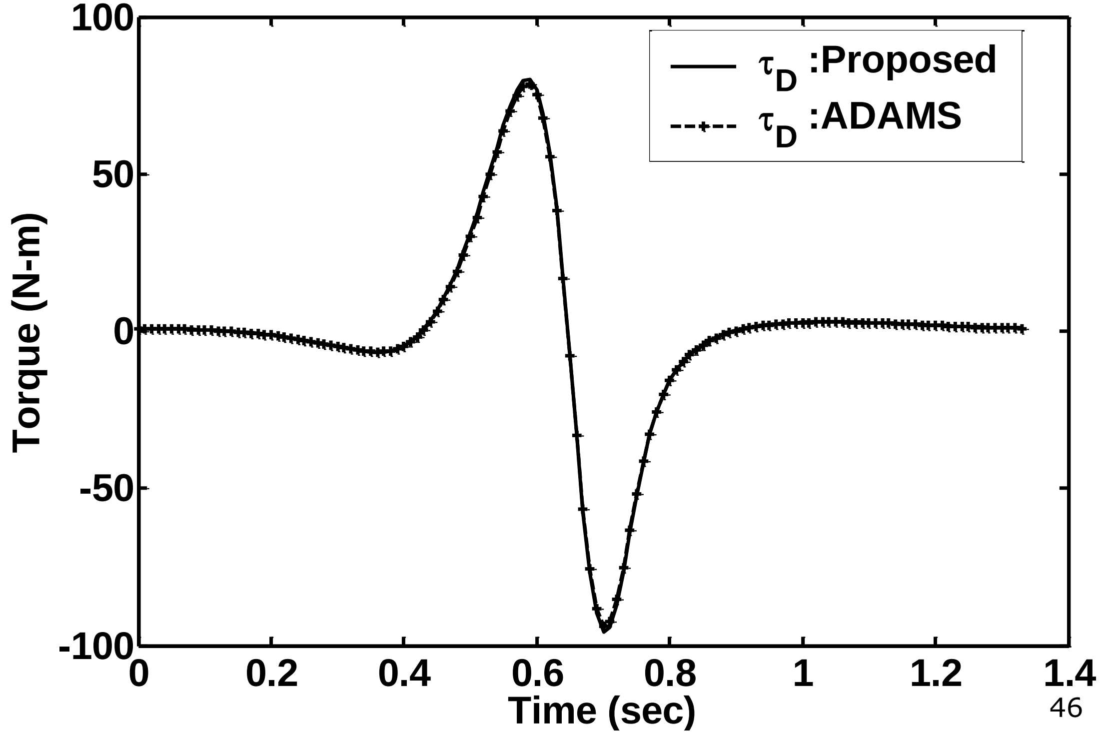


- Unknowns: 6+3 & Eqs. 2+1
- Unsolvable independently (Indeterminate subsystems)

- Unknowns: 4 & Eqs.: 4
- Solvable independently (determinate subsystem)

Results





ADAMS Animation

2009

¥10,000

Preface

This book has evolved from the passionate desire of the authors in using the modern concepts of multibody dynamics for the design improvement of the machineries used in the rural sectors of India and The World. In this connection, the first author took up his doctoral research in 2003 whose findings have resulted in this book. .... (contd.)

Conclusions

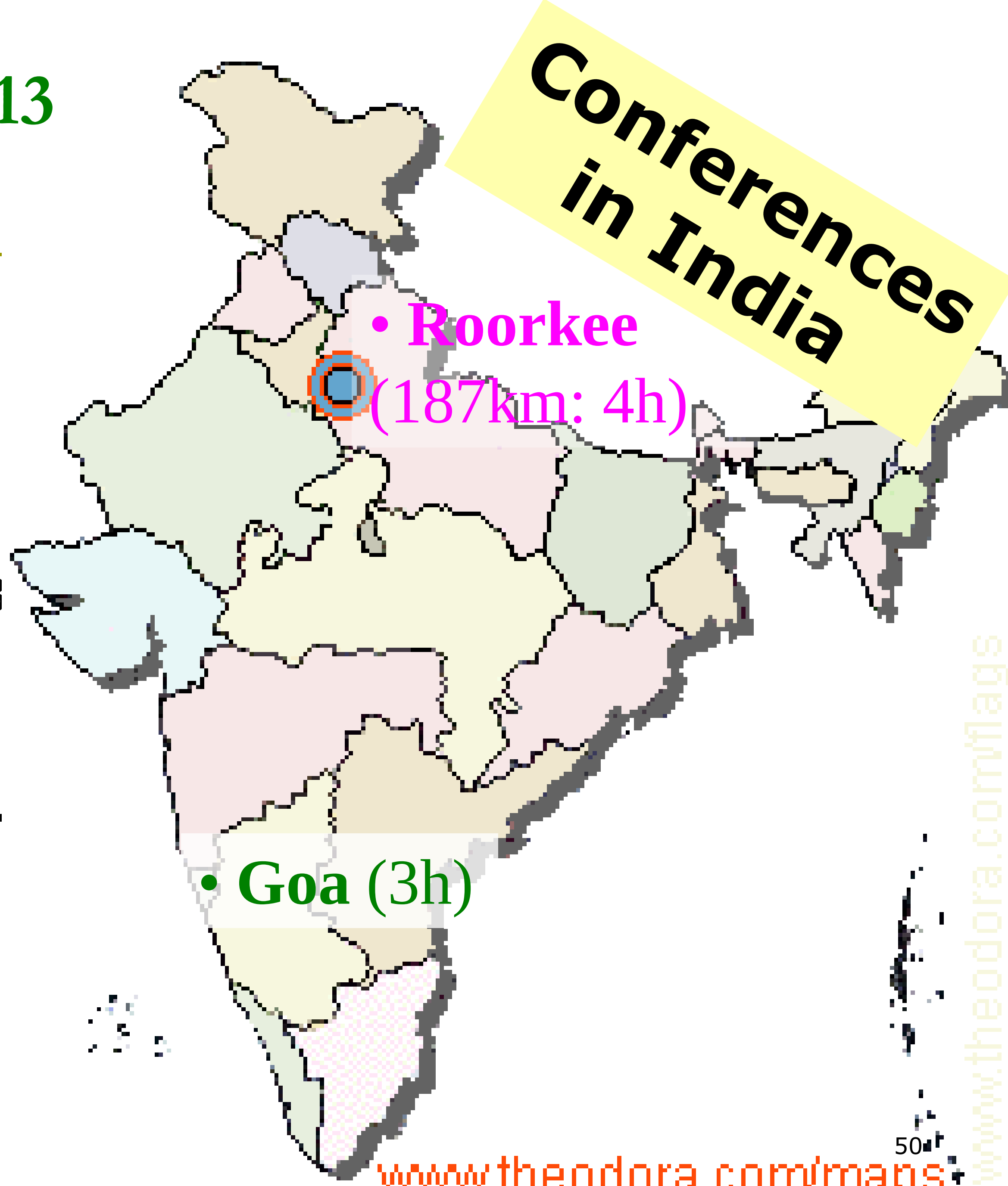
- DeNOC for serial-chain systems
- RoboAnalyzer
- Tree-type and Closed-loop systems
- Use in MuDRA

Acknowledgements

Thanks to our spouses and kids for their supports

iNaCoMM 2013
AiR-2015

- 1st Int. & 16th Nat. Conf. on Machines and Mechanisms
  - Dec. 18-20, 2013
  - IIT **Roorkee**
- Advances in Robotics: 2nd Int. Conf. of Robotics Society of India
  - July 1-4, 2015
  - **Goa**



THANK YOU

ありがとうございました

http://web.iitd.ac.in/~saha

saha@mech.iitd.ac.in

Parallel Robots
(SDD'08-10)