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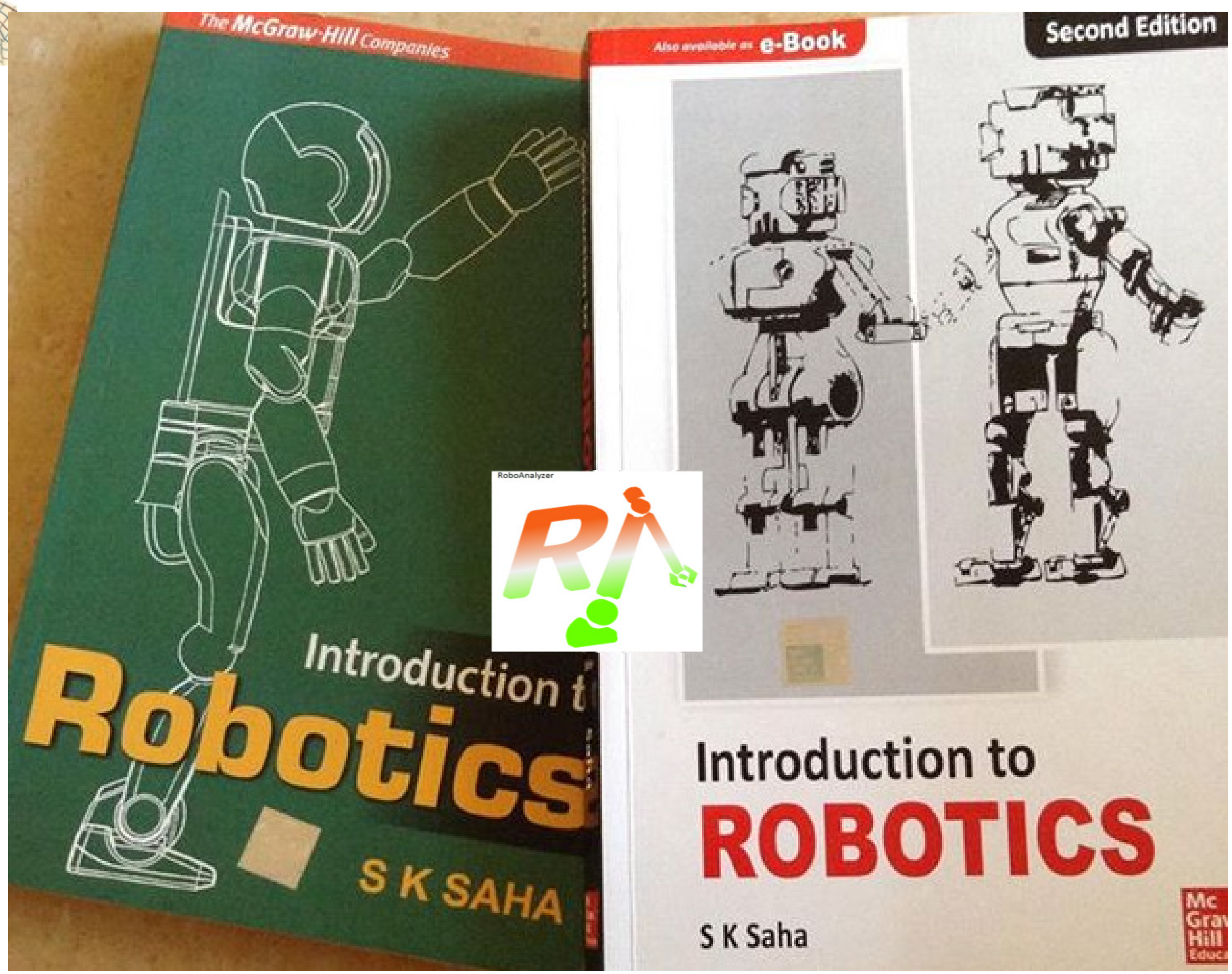
Lecture 06

Inverse Kinematics



VLFM Course Module on Robotics

February 12, 2019

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Summary of Lecture 5

- Forward kinematics methodology
  - Unique solution
- Examples
- Graphical motion to show multiple solutions for inverse kinematics

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Outline

- Review of Forward Kinematics
- Inverse kinematics
  – Multiple solutions

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Inverse Kinematics: Two-link Arm

$$p_x = a_1 c_1 + a_2 c_{12}$$

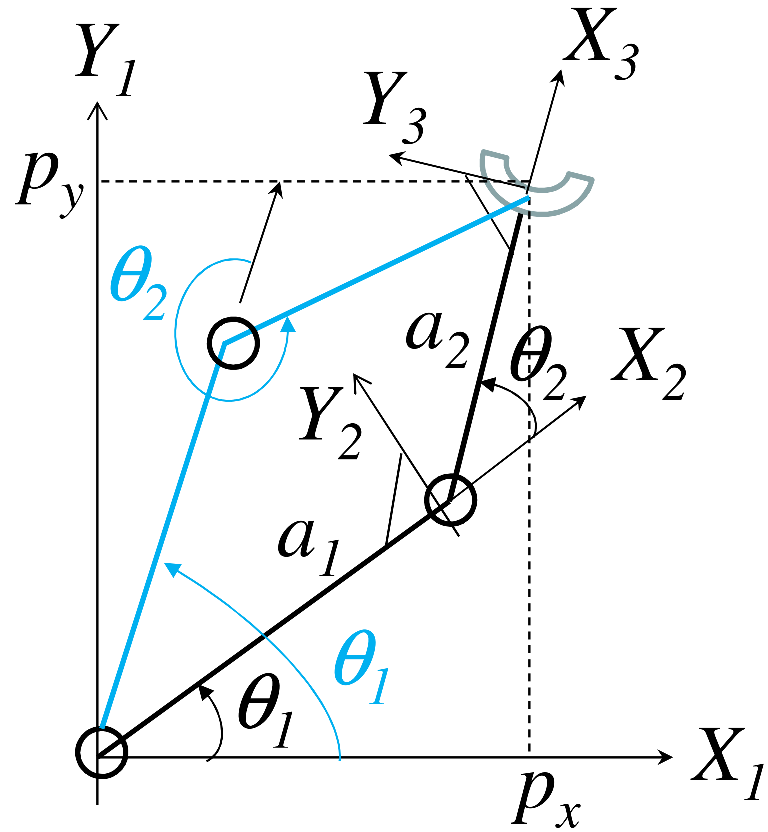
$$p_y = a_1 s_1 + a_2 s_{12}$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$s_2 = \pm \sqrt{1 - c_2^2}$$

Verify it!

$$\theta_2 = \text{atan2}\,(s_2, c_2)$$



RoboAnalyzer

$$s_1 = \frac{(a_1 + a_2 c_2)\, p_y - a_2 s_2 p_x}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = p_x^2 + p_y^2$$

$$c_1 = \frac{(a_1 + a_2 c_2)\, p_x + a_2 s_2 p_y}{\Delta}$$

$$\theta_1 = \text{atan2}\,(s_1, c_1)$$

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Numerical Example

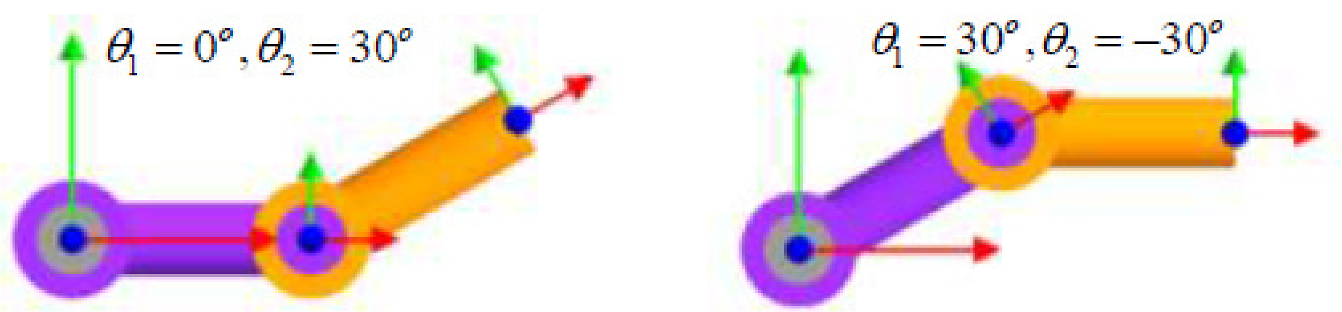
- Inverse Kinematics of Two-link Planar Arm (Example 6.14)

Input:

$$\mathbf{T} \equiv \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 1+\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse kinematics solution for the 2-link planar arm using RoboAnalyzer is given below:

Verify!



(a) Elbow down (b) Elbow up

Figure 6.9 Inverse kinematics solution of two-link planar arm

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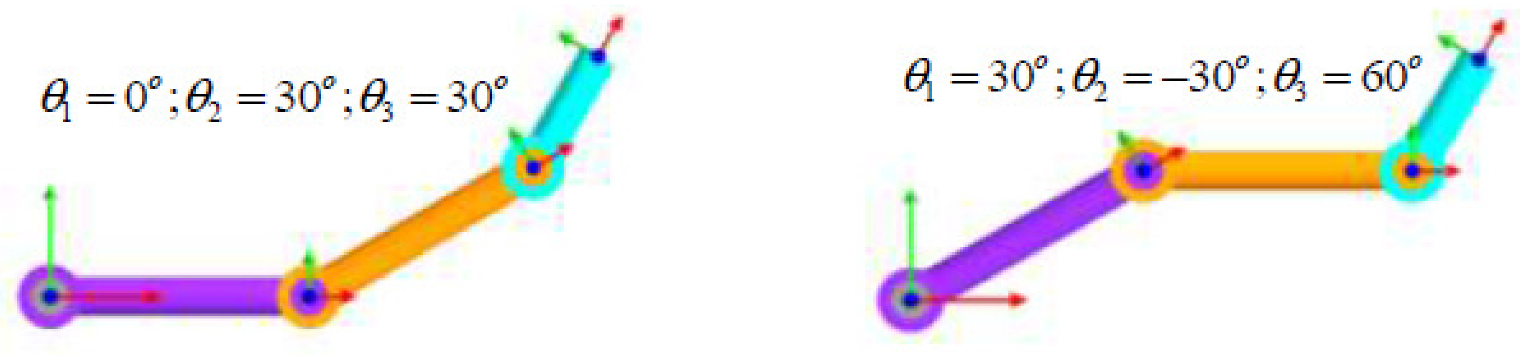
Numerical Example

- Inverse Kinematics of the Three-link Planar Arm (Example 6.15)

Input:

$$\mathbf{T} \equiv \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & \sqrt{3}+\frac{5}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse kinematics solution for the 3-link planar arm using RoboAnalyzer is given below:



(a) Elbow down (b) Elbow up

Figure 6.10 Inverse kinematics solution of three-link planar arm

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Three-DOF Articulated Arm

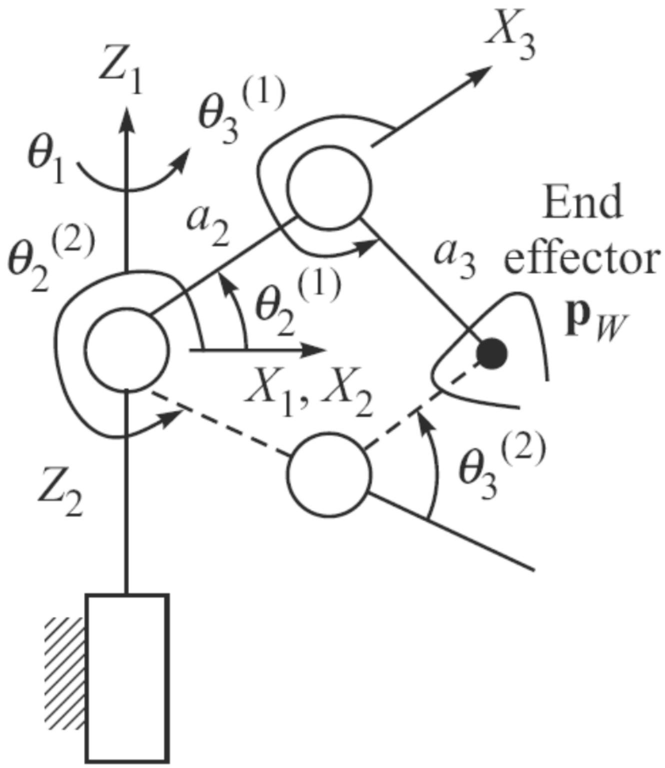
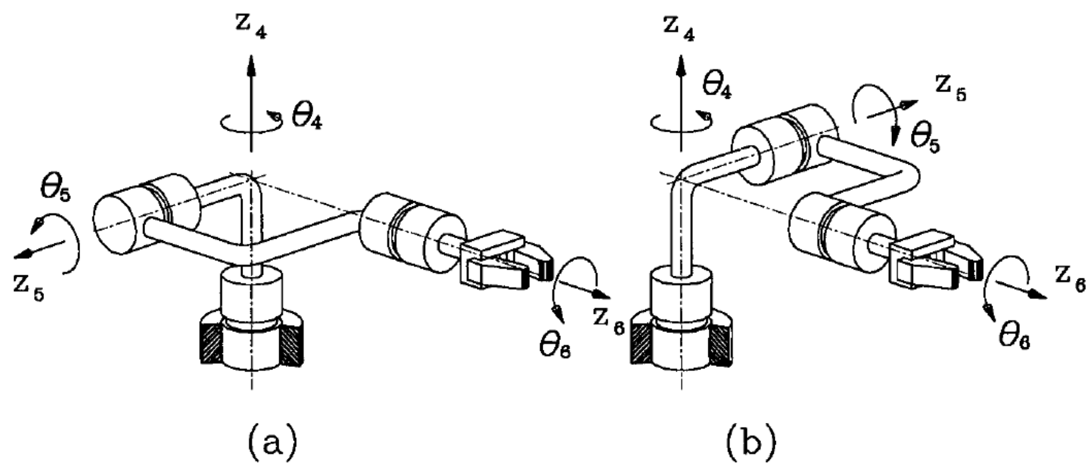


Fig. 6.7 Two admissible solution

4 solutions!

Three-DOF Wrist



2 solutions!

KUKA has 4x2=8 solutions

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Conclusions

- Multiple solutions for Inverse kinematics
- Illustration with RoboAnalyzer software

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THANK YOU

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