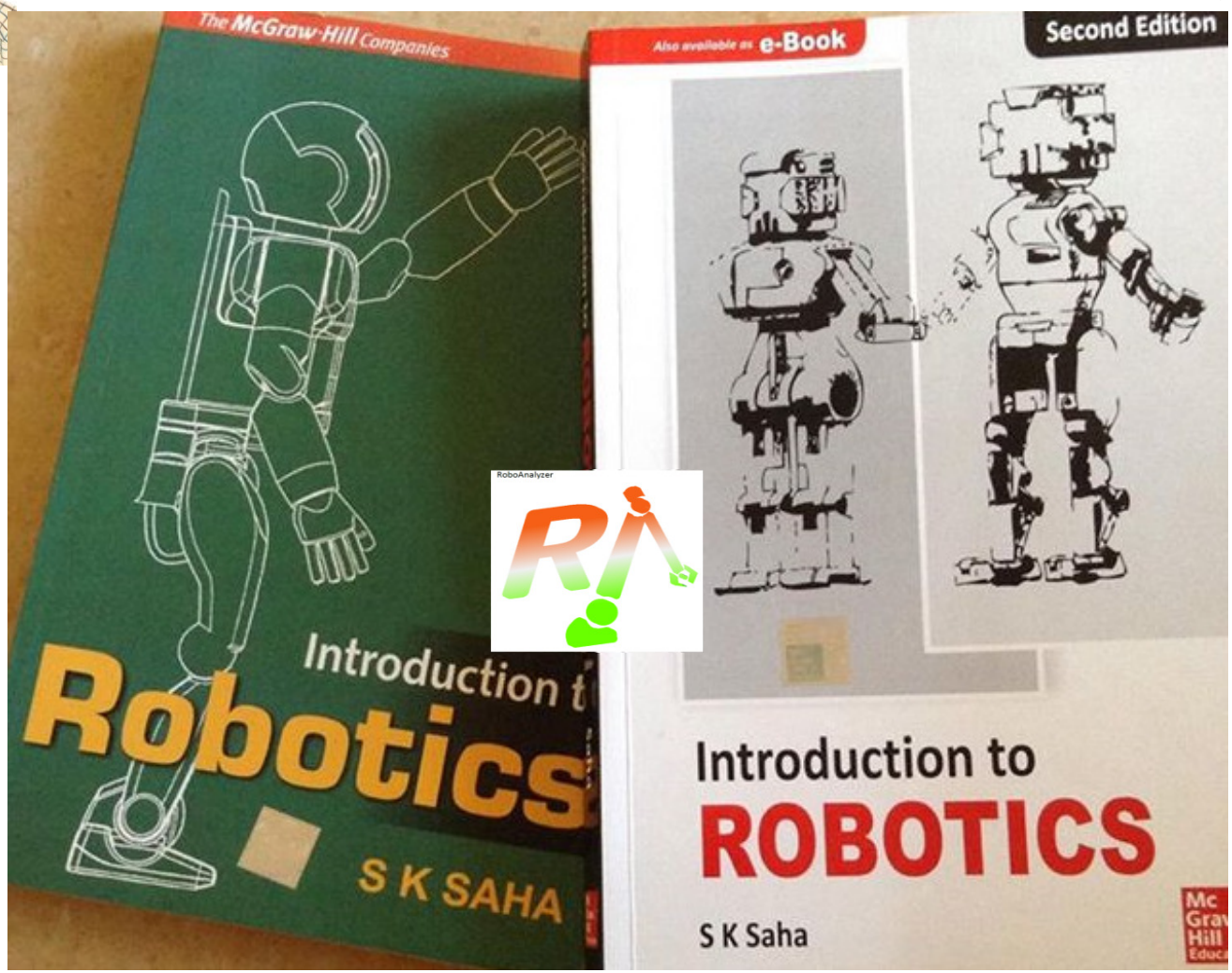
Lecture 05

Forward Kinematics



VLFM Course Module on Robotics

February 11, 2019

Announcement

- Lectures 1-4 are available in

  http://sksaha.com/courses

Review of Lecture 4

- **Coordinate Transformation**
  - **Relation between two coordinate frames**
  - **Homogeneous Transformation Matrix (HTM)**
- **Forward kinematics**
- **Use of RoboAnalyzer**

Outline

- Examples for Forward Kinematics
- Inverse kinematics
  - Multiple solutions

Kinematics

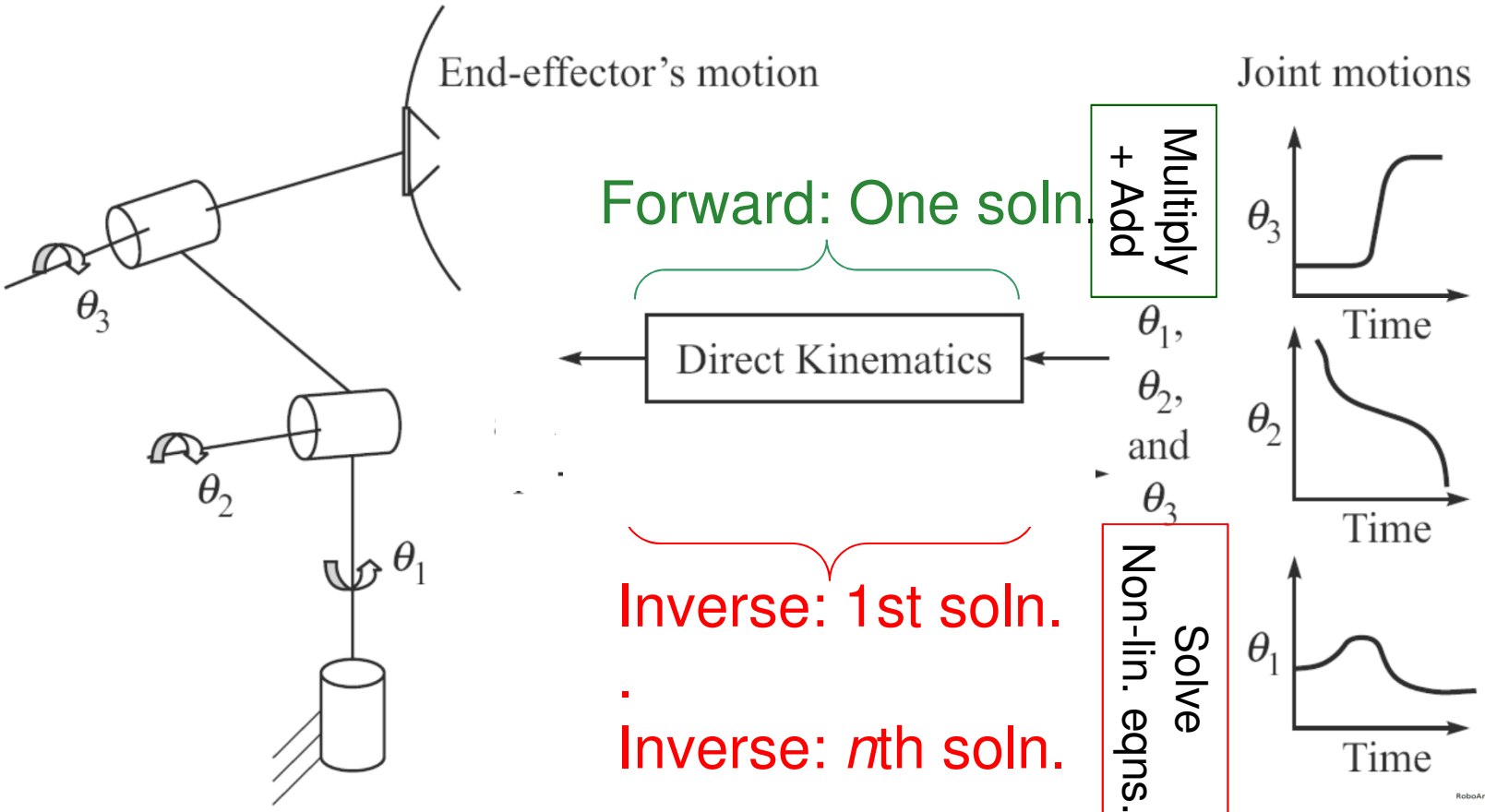


Fig. 6.1 Forward and inverse kinematics

- Forward kinematics relation

$$\mathbf{T} = \mathbf{T}_1 \mathbf{T}_2 \ldots \mathbf{T}_n$$

$$\mathbf{T}_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & \\ 0 & S\alpha_i & C\alpha_i & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix

Position

DH Parameters and Matrices for 2-link Planar Arm

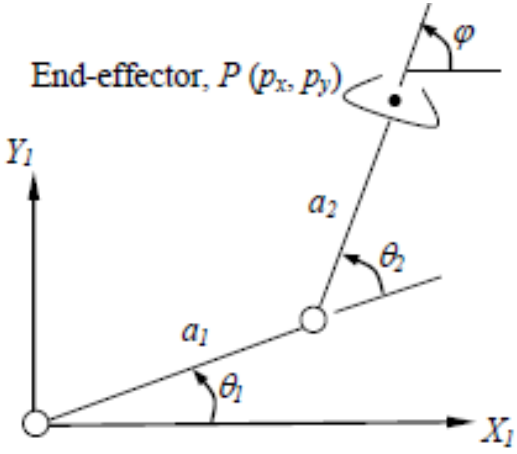


Figure 6.2 Kinematics of a two-link planar arm

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	a_1	0
2	0	θ_2 (JV)	a_2	0

For $i = 1,2$

$$\mathbf{T}_i \equiv \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{T} = \mathbf{T}_1 \mathbf{T}_2$ ⇒

$$\mathbf{T} \equiv \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ..(6.5)$$

Revolute-Prismatic Planar Arm

Non-intersecting joint axes

- DH-parameters

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	a_1	$\pi/2$
2	b_2 (JV)	0	0	$-\pi/2$

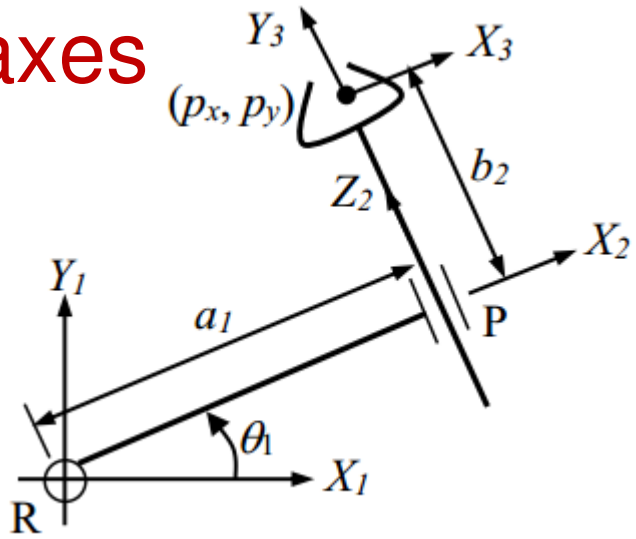


Fig. 5.30 (b) Revolute-Prismatic planar arm

- Frame transformations (Homogeneous) . . . (5.63b)

$$\mathbf{T}_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & a_1 C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & a_1 S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Examples

RoboAnalyzer

Conclusions

- Examples of Forward Kinematics
- Illustration with RoboAnalyzer software

THANK YOU

saha@mech.iitd.ac.in

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